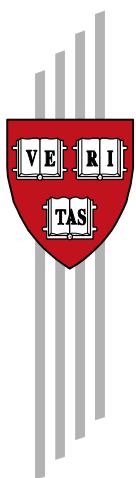


On Globalization and the Concentration of Talent

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On Globalization and the Concentration of Talent*

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Abstract

We analyze how globalization affects the allocation of talent across competing teams in large matching markets. Assuming a reduced form of globalization as a convex transformation of payoffs, we show that for every economy where positive assortative matching is an equilibrium *without* globalization, it is also an equilibrium *with* globalization. Moreover, for some economies positive assortative matching is an equilibrium *with* globalization but not *without*. The result that globalization promotes the concentration of talent holds under very minimal restrictions on how individual skills translate into team skills and on how team skills translate into competition outcomes. Our analysis covers many interesting special cases, including simple extensions of Rosen (1981) and Melitz (2003) with competing teams.

Keywords: competing teams, globalization, inequality, matching

JEL Classification: C78, D3, D4, F61, F66

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1 Introduction

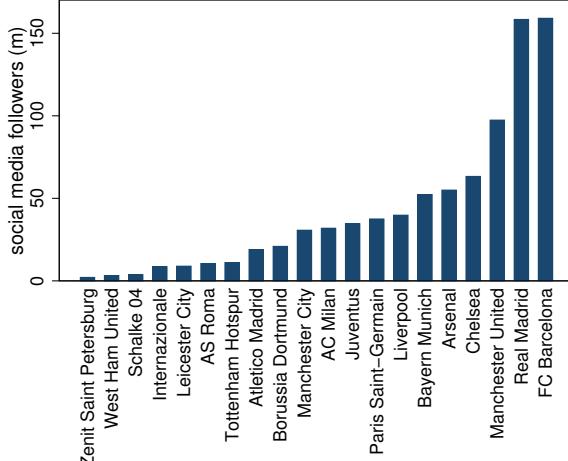
There are few topics that have received more attention in recent years than the rise of inequality in the US as well as in other parts of the industrialized world. Many debates revolve around income and wealth inequality and their driving forces, including the role played by globalization. Yet, there are other aspects to inequality (and to globalization) that deserve attention. Our departing point is the observation put forward by recent research that the rise of income inequality went hand-in-hand with a stronger segregation of the labor force. That is, increasingly over time, high-skilled (low-skilled) workers co-work with other high-skilled (low-skilled) workers (Card et al., 2013; Song et al., 2019). Higher segregation might have important consequences for income inequality, political stability, and social cohesion in general. Despite its relevance, however, this topic has received less attention in the economics literature, and the causes and consequences of a higher segregation of the labor force are not well understood yet. In this paper, we seek to contribute to such understanding by looking at how globalization impacts the concentration of talent among competing teams.

Globalization provides firms with access to new markets. These opportunities, however, do not benefit all firms alike. The most productive firms are the ones that export (Melitz, 2003; Bernard et al., 2007). Similary, only the best artists or sport teams are able to reach out to global audiences. Social media followers of football (soccer) clubs, to name one illustrative example, are highly concentrated on the most famous clubs even among the select group of clubs with highest revenues worldwide—see Figure 1. Teams (or, in general, firms) will reach out to foreign markets only if this is profitable. The fact that the best teams serve global markets therefore implies that with globalization total payoffs are more concentrated in the hands of market leaders. We argue that, as a consequence, globalization promotes the concentration of talent. That is, high-skilled (low-skilled) workers form teams with other high-skilled (low-skilled) workers more often with globalization than without.

Model

To elaborate, we build on Chade and Eeckhout (2019) and consider an economy where workers with heterogeneous skills form competing teams. We then analyze how matching outcomes are affected by globalization. In the baseline model, there are two skill levels (high and low), teams are composed of two workers, and competition among teams results in a rank distribution for each team. The rank distribution is a function of the team's own skill level (i.e., of the skill levels of its team members) as well as of the skill levels of all other

Figure 1: Social media followers of top football teams



Notes: Own illustration, based on Deloitte (2017). This figure shows the total number of social media followers for each of the 20 football clubs with highest revenues worldwide in 2015/16. This total refers to the sum of followers on Twitter and Instagram, and ‘likes’ on Facebook.¹

teams. A team’s rank in the market determines its payoff. Later, we show that our main results readily extend to the case with more skill types and/or more team members, as well as to setups with skill-dependent payoffs, as opposed to rank-dependent payoffs.

As is standard, there is an equilibrium with positive assortative matching if the teams’ expected payoffs satisfy a supermodularity condition: the expected return of a *mixed* team—i.e., a team consisting of one high-skilled worker and one low-skilled worker—cannot be larger than the average return of the teams consisting of workers of only one skill level. In our model, however, this condition depends on non-trivial interactions between how individual skills translate into teams’ skills, how teams’ skills translate into their success in the competition stage, and how success translates into payoffs. Loosely speaking, it is beneficial to pool talent if either competition outcomes are themselves supermodular—which implies that, on average, a low-skilled and a high-skilled team reach a higher rank than a mixed team—or if rewards for being a market leader are very high. In the latter case, an even marginally higher probability for positively assorted teams to reach extreme ranks can suffice to promote that high-skilled workers partner up with other high-skilled workers, no matter the degree of complementarity between skills. This means that in our setup, complementarity at the skill level—a common assumption in literature—is neither necessary nor sufficient for positive assortative matching to be an equilibrium.

¹The basic pattern is the same when merely considering ‘likes’ on Facebook.

Main results

Our main contribution is that we show that globalization—defined as a convex transformation of payoffs—will increase the concentration of talent under minimal restrictions on the effect of skills on competition outcomes.² Specifically, we show that in any economy where positive assortative matching is an equilibrium *without* globalization, it must also be an equilibrium *with* globalization.³ Moreover, there are economies where positive assortative matching is an equilibrium *with* globalization but not *without*. The opposite is true for equilibria with negative assortative matching. More generally, we show that the equilibrium with highest concentration of talent arises with globalization while the equilibrium with lowest concentration of talent arises without globalization. The basic intuition is simple: teams whose members are matched positively assorted are—on average—more likely to reach the highest ranks, the ones that benefit most from globalization.

The increase in the concentration of talent has important distributional consequences, and a globalization-induced change in the equilibrium matching has an additional effect on relative wages when compared to the case where workers are always matched positively assorted. The latter case is typically considered in the literature. In fact, we show that under reasonable assumptions on how the success of a low-skilled team changes with the matching in the overall economy, a globalization-induced increase in the concentration of talent adds to income inequality.⁴

Our main result—globalization promotes positive assortative matching—is remarkably general. With regard to the competition mode among teams, we simply assume that mixed teams are more likely to be ranked in the mid range, with teams formed only by high-skilled (low-skilled) individuals more likely to be ranked in the upper range (lower range). This puts minimal restrictions on the mode of competition and renders the relationship between skills and performance meaningful. As to the payoff scheme, we just apply the normalization that higher ranks are better, i.e., we assume that the payoff scheme is increasing over ranks. Moreover, while for expositional purposes we consider a baseline setup consisting of two

²Hence, we focus on one particular aspect of globalization, namely its role as an amplifier of ‘superstar-effects’ à la Rosen (1981). This is an important aspect of globalization—see the literature review in Section 2, as well as our discussions above and in Section 7.

³Our paper is about comparative statics, and in particular our model is not dynamic in nature. This means that, properly speaking, we should say “without/with” globalization. However, as is well documented and is discussed in more detail in Sections 2 and 7.2, globalization has taken effect in the last decades. Hence our use at times of “before (after) globalization” instead of “without (with) globalization”.

⁴An increased concentration of talent may well have distributional consequences above and beyond any immediate effect on wages, e.g. in the presence of knowledge spillovers. We briefly allude to such matters in the conclusion and leave a thorough investigation for future research.

types of workers (low- and high-skilled) who form teams of two and compete in a market with rank-dependent payoffs, we later show that our results readily extend to the case of $S \geq 2$ types and $N \geq 2$ team members, as well as to setups with skill-dependent payoffs.

The weak nature of our assumptions has the advantage that our reduced-form analysis covers many interesting special cases. On the one hand, we show that mixed teams are naturally more likely to achieve mid-range ranks in head-to-head competitions or patent races, for example, as well as in situations with skill-dependent productivity draws from a Pareto distribution. On the other hand, globalization acts as an amplifier of superstar effects in the Rosen (1981) model and the Melitz (2003) model with fixed cost of market entry, for example. Hence, our results directly apply to simple extensions of these models encompassing competing teams.

Illustrative example: The case of European football

Our main mechanisms of interest can be illustrated by the competition in European football leagues. European football is particularly well suited for our purposes for the following reasons: First, football teams compete in their national leagues for rank. Second, a team's performance in these leagues is a direct measure for its skill level relative to the skill levels of the competing teams. Third, the increasing importance of UEFA Champions League is a prime example for our concept of globalization, as we now explain. Yet, our paper is not about football per se, and we argue that our theory is informative for any environment with competing teams, including, for example, research, sports, entrepreneurship, and consulting.

In European football, clubs compete in national leagues. Each year, the clubs ranked highest in their respective national league qualify to participate in the UEFA Champions League. This is a pan-European competition that was transformed in the course of the 1990s from a knock-out competition between national champions into something close to a European super-league today, and it has witnessed enormous growth in the past. Annual payouts to participating teams, for example, increased more than twentyfold since 1996/97, reaching more than EUR 2bn in 2018/19. Moreover, the final of the UEFA Champions League is broadcasted in over 200 countries, with up to 400m people tuning in, making it the biggest annual sports-event worldwide.⁵ The Champions League is so dominant today that qualification for it is sometimes considered to be as important as winning even the

⁵See https://www.footballbenchmark.com/uefa_champions_league_non_big_five_participation, <https://www.uefa.com/uefachampionsleague/news/newsid=2562033.html>, and <http://fr.uefa.com/uefachampionsleague/news/newsid=2111684.html#/> (retrieved on 31 December 2017).

most prestigious national leagues.⁶

The establishment and growth of the UEFA Champions League is a prime example of globalization in a market. How does globalization feed back into competition in national leagues? The Champions League provides participating teams with large direct payouts and, in addition, with a global platform—in particular those teams that frequently proceed to the final rounds in this competition.⁷ As a consequence of the increasing importance of the UEFA Champions League, the returns to being ranked high in a national league have increased, reflecting amplified ‘superstar effects’. The key observation is that this development went hand in hand with an increased concentration of talent, as may be seen from Figure 2(a). This figure shows the share of the maximal achievable points won by the respective national champion for each of the ‘big 5’ European football leagues—England, France, Germany, Italy, and Spain. This share has been steadily increasing over time, suggesting that the players with the highest talent are increasingly concentrated in a few teams (viz., those who win). In fact, the increased concentration of talent is currently also a matter of concern for officials at the Fédération Internationale de Football Association (FIFA).⁸

To further substantiate our conjecture that this increase in the concentration of talent is attributable to globalization, we contrast our ‘treatment group’ of first-division leagues with the ‘control group’ of corresponding second-division leagues—see Figure 2(b). Teams in these leagues cannot qualify for the UEFA Champions League (or any other European competition) via their national leagues, so globalization is less important for competition in these leagues. If globalization was a key driver for the concentration of talent, we should not observe the same upward trend in second-division leagues. As shown in Figure 2(b), this is indeed the case.

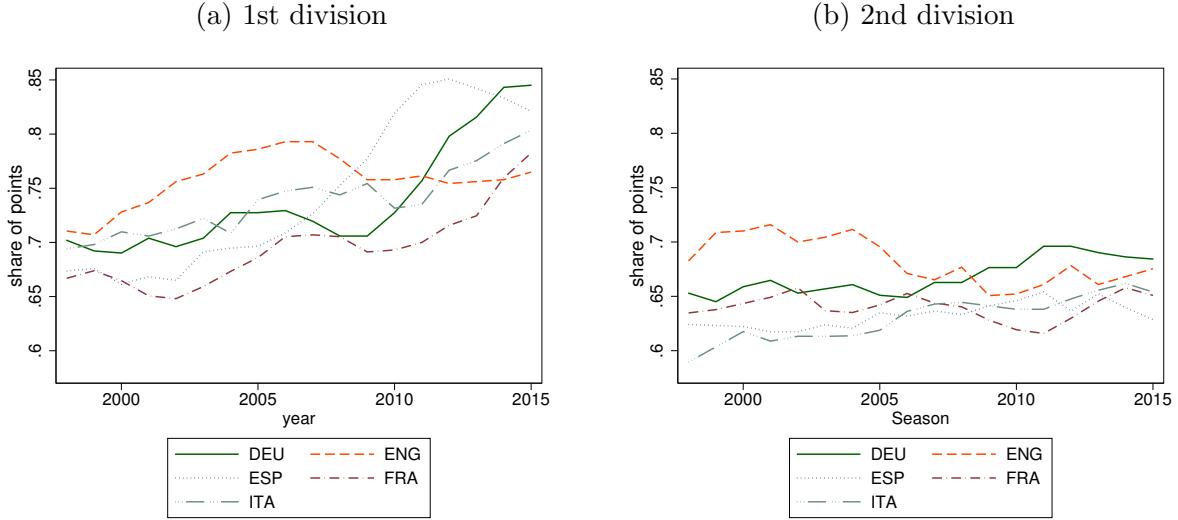
In summary, the data reveals an increasing-over-time concentration of talent in European football and suggests that this may be attributable to globalization. In the remainder of the paper we show that globalization indeed promotes the concentration of talent. We argue that this is, in fact, not only the case in European football, but that similar mechanisms apply in different contexts, and, in particular, in any setup with competing teams and where

⁶See for example <http://www.telegraph.co.uk/sport/football/teams/arsenal/9633456/Arsenal-manager-Arsene-Wenger-says-qualifying-for-Champions-League-on-a-par-with-winning-a-trophy.html> (retrieved on 31 December 2017).

⁷See Figure 1. Real Madrid and FC Barcelona, for example, are the teams with most semi-final appearances from 2000 to 2017 (11 and 10, respectively).

⁸See, e.g. <http://www.espn.com/soccer/fifa-world-cup/story/3383510/fifa-president-gianni-infantino-backs-sweeping-transfer-market-reform> (retrieved on February 18, 2018).

Figure 2: Concentration of talent in European football leagues



Notes: Own illustration, based on kicker.de and wikipedia.org. The share of points refers to the centered 5-year moving average of the ratio of end-of-season points of the national champion over the maximum achievable number of points. Panel (a) refers to the respective first-division league for each of the five countries, panel (b) to the corresponding second-division league. We thank Stefan Legge for sharing the data on second-division outcomes.

globalization favors market leaders. We return to this point in Section 7, where we discuss micro-foundations for our main assumptions.

Organization of the paper

The remainder of the paper is organized as follows. In Section 2 we review the different strands of literature that are related to our paper. In Section 3 we introduce the baseline version of our model. In Section 4 we analyze equilibria in our economy. In Section 5 we investigate the effect of globalization on equilibrium outcomes. In Section 6 we consider extensions with several types and team members, alternative modes of competition, and migration. In Section 7 we present microfoundations for our main assumptions and show that our reduced-form analysis covers interesting special cases. Section 8 concludes. The proofs of all the results are in the Appendices.

2 Relation to the Literature

In our model, globalization increases the gains from being ranked high in a market, i.e. we think of globalization as an amplifier of superstar-effects. In his seminal contribution, Rosen (1981) shows how small differences in the talent of entertainers can result in large

heterogeneity in their income if revenues are a convex function of talent. He argues that this is particularly true in markets with imperfect substitutability between artists of different quality and when the marginal cost of reaching out to additional customers is low or even zero as, for example, with performances broadcasted on TV. In the original work by Rosen (1981), superstars benefit from being able to reach broader audiences.⁹ As long as consumption is indivisible in the sense that an increase in quantity cannot compensate for a lower quality, similar effects can, however, also arise if suppliers can serve a fixed number of clients only. Then, increased income inequality on the side of the buyers can translate into income inequality for suppliers. Such mechanisms can explain the increased dispersion in house prices (Määttänen and Terviö, 2014), imply that inequality can spill over across occupations (Clemens et al., 2017), and they give rise to higher CEO pay in a globalized world with larger firm sales (Gabaix and Landier, 2008; Terviö, 2008; Gersbach and Schmutzler, 2014; Ma and Ruzic, 2017). These papers have in common that there is always positive assortative matching between buyers' income (or firm size) and suppliers' quality. They carry out comparative statics exercises that can be linked to globalization, keeping the matching constant. While we also consider comparative statics with regard to globalization that strengthens superstar effects, our model and the main focus of our analysis are very different. We do not consider matching between buyers and sellers, but between workers who form competing teams. We then study the conditions under which positive assortative matching arises and show that globalization increases the concentration of talent. In turn, this may fuel (top-) income inequality. Our paper thus focuses on the analysis of a complementary channel through which globalization can add to income inequality via superstar effects.¹⁰

We build on the literature characterizing matching equilibria. In his seminal contribution, Becker (1973) showed that there will be positive assortative matching in a marriage market whenever a couple's payoff function is supermodular in the partners' types (characteristics).¹¹ In his paper, payoffs depend exclusively on the own matching, and thus they do not depend on the matching of all other men and women in the market. By contrast, we

⁹Haskel et al. (2012) discuss how globalization can amplify superstar-effects and argue based on an augmented Heckscher-Ohlin model that superstar-effects à la Rosen (1981) may well have contributed to recent trends in US wage inequality.

¹⁰Our work thus also relates to the broader literature analyzing different drivers of (top-)income inequality, see e.g. Piketty et al. (2014); Bénabou and Tirole (2016); Jones and Kim (2015); Geerolf (2017) for recent theoretical contributions. It also relates, though less closely, to empirical work by Neffke (2019), who analyzes wage effects of coworker matching using Swedish data on detailed educational attainments.

¹¹See Kremer (1993); Shimer and Smith (2000); Legros and Newman (2002, 2007); Eeckhout and Kircher (2018) for extensions of these ideas and conditions for positive assortative matching in different contexts.

assume that each team's (expected) payoff depends on the skill levels of all other teams. We borrow from Chade and Eeckhout (2019), who study matching outcomes in a large one-sided market with competition between teams.¹² They show that in such cases, multiple equilibria may exist and equilibria need not be efficient. Our focus is different. We are concerned neither with uniqueness nor with efficiency of equilibrium outcomes, but with analyzing how globalization impacts matching outcomes.¹³ To that end, we consider generic forms of competition and summarize this competition in a rank distribution function for each team that depends on a team's own skill level and the skill levels of all other teams in the market. We then apply a convex transformation of the payoff function that we interpret as a reduced-form representation of globalization.

Finally, our work is also related to the literature analyzing the distributional consequences of globalization more generally. A large literature focuses on international trade. In recent work, trade has been shown to have heterogeneous effects across regional labor markets (e.g. Autor et al. 2013; Dauth et al. 2014; Dix-Carneiro and Kovak 2015), across (types of) workers (e.g. Autor et al. 2014; Galle et al. 2015; Lee 2016; Helpman et al. 2017), and across (types of) consumers (e.g. Faber 2014; Fajgelbaum and Khandelwal 2016). The work by Costinot and Vogel (2010) is somewhat closer to our paper. They consider an assignment model of heterogeneous workers to tasks to study the distributional consequences of international trade. In their model, however, there is always positive assortative matching of workers to tasks. Grossman et al. (2016) consider two-sided matching between managers and workers of different skills that sort into various industries and analyze the distributional effects of changes in the trade environment. While in their setup, workers and managers always match in a positively assortative fashion within industries, they may or may not sort in the same fashion across industries, i.e., talent may or may not concentrate in one industry. Grossman et al. (2016) do not consider how this concentration itself is affected by globalization, which is our main focus. In this vein, our paper is closest to Helpman et al. (2010) and Porzio (2017). These papers are nonetheless very different from ours in terms of the economic environment and the main mechanisms of interest.¹⁴

¹²Another strand of this literature studies existence conditions for stable matchings and efficiency of these matchings in two-sided markets with externalities (Sasaki and Toda, 1996; Hafalir, 2008; Mumcu and Saglam, 2010; Pycia and Yenmez, 2017).

¹³In our baseline setup, we consider the case of pure competition for rank. With total payoff in the market being independent of matching, equilibrium outcomes are trivially efficient.

¹⁴Helpman et al. (2010) show how after trade liberalization, the concentration of talent in the most productive firms increases in a Melitz (2003)-model with search frictions in the labor market. Porzio (2017) shows how globalization—in his case, the availability of state-of-the-art technologies in developing countries—can give rise to a dual economy where high-skilled individuals concentrate in the sectors that adopt the state-of-the-art technology.

3 Baseline Model

To analyze how globalization impacts the concentration of talent, we build on Chade and Eeckhout (2019) and consider an economy in which workers of different skills form competing teams. In our baseline model, teams compete for rank, and each rank is awarded a payoff according to some payoff scheme. We then apply a convex transformation of this payoff scheme—to be interpreted as a reduced form of globalization—, and analyze whether as a result of such transformation, positive assortative matching is an equilibrium for a broader set of competitions. In the present section, we introduce the baseline version of our model, which entails two types of workers and teams made up of two workers. In Section 6 we show how our main result extends to the case with $S \geq 2$ types and $N \geq 2$ team members, as well as to setups with skill-dependent as opposed to rank-dependent payoffs. In what follows, we formally describe our model and its main underlying assumptions, which are further justified in Section 7.

3.1 Economic environment

The economy is populated by a continuum of measure two of workers, denoted by \mathcal{W} . Workers receive linear utility in money and differ in their skills: they are either high-skilled or low-skilled. To simplify the exposition, for now we assume that there is an equal share of each type, but this is not essential—see Section 6.1. All workers of the same type are otherwise indistinguishable from each other. We let $\mathcal{W} = \mathcal{W}^l \cup \mathcal{W}^h$, where $\mathcal{W}^h = \mathcal{W}^l = [0, 1]$ denote the set of high-skilled and low-skilled workers, respectively. Workers match to other workers and form *teams*, which consist of a pair of workers and are generally denoted by t . A team may be of three types: two high-skilled workers may match, and we use t^h to denote such a team; two low-skilled workers may match, and we use t^l to denote such a team; one worker of each type may match, and we use t^m to denote such a team. Teams potentially differ in their overall skill level, which is weakly increasing in the skills of each team member. Side-payments are possible, and thus we consider an environment with transferable utility.

A *matching* μ is the collection of all teams. That is, each worker belongs to exactly one team, and thus a matching partitions the set \mathcal{W} . We denote the set of all possible matchings of \mathcal{W} by \mathcal{M} .¹⁵ Except for changes that affect sets of workers of measure zero, \mathcal{M} can be indexed in our setup by $\mu(\alpha)$ with $\alpha \in [0, 1]$, where $(1 - \alpha)$ denotes the fraction of all

¹⁵For the sake of notation, we drop the dependence of the set of all possible matchings on \mathcal{W} .

teams in $\mu(\alpha)$ that are t^m . In turn, $\alpha/2$ denotes the fraction of all teams that are t^h and t^l , respectively. Because we pay particular attention to the case where $\alpha = 1$, we accordingly refer to $\mu(1)$ as the *positive assortative matching (PAM)*. Similarly, we refer to $\mu(0)$ as the *negative assortative matching (NAM)*.

3.2 Competition between teams

Teams compete for rank, since their payoff depends only on their rank $y \in [0, 1]$. We apply the convention that rank $y = 1$ ($y = 0$) is the best (the worst). The exact nature of the competition does not matter for the purpose of the paper, and for now it suffices to assume that competition results in some rank distribution for each team. This rank distribution depends on a team's own skill level, as well as on the skill levels of all other teams in the economy. The latter can be summarized by the share of teams whose members are matched positively assorted, namely α . We use $F^{\alpha,k}(y)$ to denote the probability that given a matching $\mu(\alpha)$, with $\alpha \in [0, 1]$, a team t^k will reach rank y or lower, with $k \in \{l, m, h\}$. In other words, $F^{\alpha,k}(y)$ is the cumulative distribution function (CDF) of team t^k over ranks $y \in [0, 1]$ when all teams are arranged according to $\mu(\alpha)$. In summary, we conceive of the competition as determining the rank distribution of each team as a function of its own skill level and the skill levels of all other teams in the economy. Accordingly, we formally define the *competition* as a set of rank distributions for each team type $k \in \{l, m, h\}$ and each matching $\mu(\alpha)$, $\mathcal{F} = \{(F^{\alpha,l}, F^{\alpha,m}, F^{\alpha,h})\}_{\alpha \in [0,1]}$. This reduced-form representation covers many interesting special cases, some of which are discussed in Section 7.1.

For simplicity, we assume that for all $\alpha \in [0, 1]$ and all $k \in \{l, m, h\}$, $F^{\alpha,k}(\cdot)$ is continuously differentiable, and at times we denote the corresponding probability distribution function (PDF) by $f^{\alpha,k}(y) := \frac{dF^{\alpha,k}(y)}{dy}$. Because all positions in the ranking have to be filled, we must have that for all $y \in [0, 1]$,

$$1 = \frac{\alpha}{2} \cdot f^{\alpha,l}(y) + \frac{\alpha}{2} \cdot f^{\alpha,h}(y) + (1 - \alpha) \cdot f^{\alpha,m}(y). \quad (1)$$

To render the notion of skill meaningful, we define $\mathcal{A}_\alpha^\mathcal{F} := \{y \in [0, 1] : f^{\alpha,m}(y) \geq 1\}$ and $\tilde{\mathcal{A}}_\alpha^\mathcal{F} := \{y \in [0, 1] : f^{\alpha,l}(y) + f^{\alpha,h}(y) \leq 2\}$ for all $\alpha \in [0, 1]$, and then consider the following two conditions:

Assumption 1 (Competition)

(i) (PAM version) The set $\mathcal{A}_1^\mathcal{F}$ is convex and compact.

(ii) (NAM version) The set $\tilde{\mathcal{A}}_0^\mathcal{F}$ is convex and compact.

Note that for all $\alpha \in (0, 1)$, we have $\mathcal{A}_\alpha^F = \tilde{\mathcal{A}}_\alpha^F$ by Equation (1). Further, if rank distributions are continuous in the distributions of skills in the entire economy, the boundaries of the sets \mathcal{A}_α^F and $\tilde{\mathcal{A}}_\alpha^F$ are continuous in α .¹⁶ Any restrictions on the sets \mathcal{A}_α^F and $\tilde{\mathcal{A}}_\alpha^F$ are then a reflection of the same underlying rationale, and we introduce this distinction to avoid technical difficulties pertaining to the limiting behavior of the various rank distributions when analyzing equilibria with positive and negative assortative matching, respectively.

For further understanding of the economic content of Assumption 1, consider the case of some $\alpha \in (0, 1)$, with the interpretation of Assumption 1(i) and Assumption 1(ii) following. Since $f^{\alpha,m}(y)$ is continuous in $[0, 1]$, the set \mathcal{A}_α^F is compact. This is a standard assumption of rather technical nature. The more interesting part of Assumption 1 is that the set \mathcal{A}_α^F is convex. This is a minimal definition of skills in our economy. It is satisfied if $f^{\alpha,m}(\cdot)$ is monotonic or quasi-concave, and in particular if it is single-peaked. More generally, it is satisfied if a mixed team is likely to achieve ranks in the mid range.

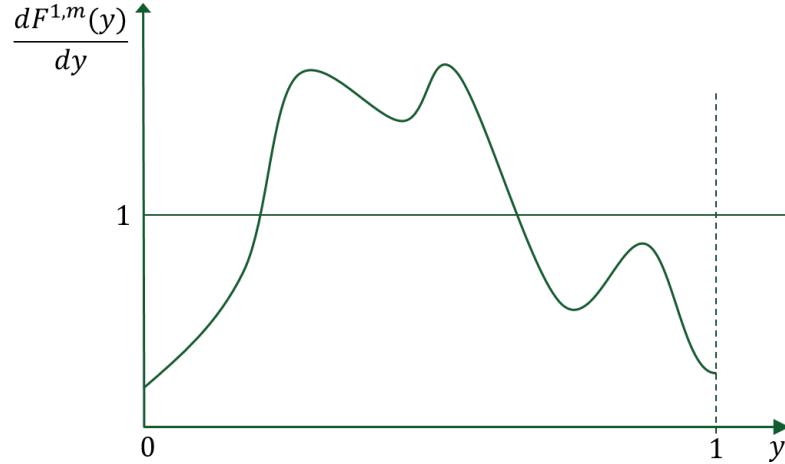
To see the above, consider that matching $\mu(\alpha)$ has formed, i.e., there are shares $1 - \alpha$ of mixed teams and $\alpha/2$ of low- and high-skilled teams, respectively. What should each of the mixed teams expect in terms of its ranking in the competition? One possibility is that $f^{\alpha,m}(y) = 1$ for all $y \in [0, 1]$, in which case $\mathcal{A}_\alpha^F = [0, 1]$ and all such teams expect to be ranked in any position with the same probability. In general, \mathcal{A}_α^F is the set of ranks whose associated probability for a mixed team is at least that of the former uniform case. This determines in itself a plausible threshold that separates ranks $y \in [0, 1]$ into *low-probability ranks* ($f^{\alpha,m}(y) < 1$) and *high-probability ranks* ($f^{\alpha,m}(y) \geq 1$).

By assuming that \mathcal{A}_α^F is a convex set, we rule out competitions whose outcome does not behave naturally with regard to the rank of the mixed teams. Indeed, if skills of the members of a team are to yield an advantage for the ranking in the competition, mixed teams should expect to be ranked in mid-range positions more often than in the uniform case, with low-skill teams being ranked low and high-skill teams being ranked high more often than in the uniform case. A weak way of implementing this rationale is to preclude the possibility that for a mixed team, a low-probability rank exists between two high-probability ranks. This is captured in Assumption 1 and illustrated by Figure 3 for the case of Assumption 1(i).

Finally, we note that Assumption 1 is naturally satisfied in many economic applications, including head-to-head competitions, patent races, and situations with random productivity draws. We document this in Section 7.1.

¹⁶Our subsequent analysis does not hinge on this restriction.

Figure 3: Illustration of Assumption 1(i)



3.3 Payoffs

Teams receive rank-dependent payoffs. We let

$$\begin{aligned} h : [0, 1] &\rightarrow \mathbb{R}_+ \\ y &\rightarrow h(y) \end{aligned} \tag{2}$$

denote the *payoff scheme* that assigns a certain amount of money $h(y)$ to the team that has been ranked in position $y \in [0, 1]$. Remaining consistent with our normalization that higher ranks are better, we assume that payoffs are weakly increasing in rank.

Assumption 2 (Payoffs)

- (i) $h(0) = 0$,
- (ii) $h'(\cdot) \geq 0$.

The normalization $h(0) = 0$ can be made without loss of generality. With $h(\cdot)$ constant, there would be no effect of workers' skills on (expected) outcomes. In what follows, we thus limit our attention to the economically interesting cases where $h(\cdot)$ is non-constant and let

$$\mathcal{H} = \{h : [0, 1] \rightarrow \mathbb{R}_+ : h \in C^1([0, 1]), h'(\cdot) \geq 0, 0 = h(0) < h(1)\} \tag{3}$$

denote the set of admissible payoff schemes, where $C^1([0, 1])$ denotes the set of continuously differentiable functions on $[0, 1]$.

3.4 Globalization

Our main focus is on the redistributive aspects of globalization and on how these interact with the concentration of talent. To that end, we consider a reduced-form modeling

choice for globalization. Specifically, we represent (the effect of) globalization by a twice continuously differentiable function

$$\begin{aligned} g : \mathbb{R}_+ &\rightarrow \mathbb{R}_+ \\ x &\rightarrow g(x), \end{aligned} \tag{4}$$

which modifies the payoff scheme and transforms it from h to $g \circ h$.¹⁷ We assume that $g(\cdot)$ satisfies the following restrictions:

Assumption 3 (Globalization)

- (i) $g(0) = 0$,
- (ii) $g'(0) > 0$,
- (iii) $g''(\cdot) \geq 0$,

Assumptions 3(i) and 3(ii) are mainly technical in nature. They simply guarantee that $g \circ h \in \mathcal{H}$ for all $h \in \mathcal{H}$ and all $g \in \mathcal{G}$, where \mathcal{G} is the set of functions satisfying Assumption 3. Assumption 3(iii) is more substantive. It implies that globalization takes the form of a convex transformation of the payoff scheme, which is also increasing thanks to Assumption 3(ii). The latter feature preserves the normalization that higher ranks are better, while the increased convexity of payoffs reflects an amplified ‘superstar-effect’, as already noted by Rosen (1981). Assumption 3 is further justified in Section 7.2.

4 Equilibrium

In the previous section we outlined the baseline version of our model. To sum up, we are considering an *economy* that is characterized by a triple $e = (\mathcal{W}, h, \mathcal{F})$, composed of a set of workers \mathcal{W} , a payoff scheme $h(\cdot)$, and a competition \mathcal{F} . We now proceed to the analysis of equilibria in each economy $e \in \mathcal{E}$, where we use \mathcal{E} to denote the set of all possible economies that satisfy our previous restrictions. We start by characterizing the potential equilibria and then discuss their general existence.

4.1 Characterization of equilibria

Given $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$, it is useful to define, for $k \in \{l, m, h\}$,

$$V(t^k | \mu(\alpha)) := \int_0^1 h(y) dF^{\alpha, k}(y). \tag{5}$$

¹⁷Our approach to modelling globalization is similar in spirit to the approach taken e.g. by Grossman et al. (2016), who model globalization—a change in the trade environment in their case—as a change in relative output prices.

That is, $V(t^l|\mu(\alpha))$, $V(t^m|\mu(\alpha))$, and $V(t^h|\mu(\alpha))$ denote the expected payoff of a team t^l , t^m , and t^h , respectively, in economy e when teams are arranged according to $\mu(\alpha)$.¹⁸ Now, remember that we are assuming linear utility in money, so agents care about their expected payoffs only. Further, with a continuum of workers, we can see them as operating in a competitive labor market. This means that workers can decide to form teams with other workers in the economy, taking as given the wage rates for skilled and unskilled workers, which we denote by \bar{w} and \underline{w} , respectively.¹⁹ An equilibrium in an economy e is therefore defined as follows:

Definition 1 (Equilibrium)

An equilibrium of economy $e \in \mathcal{E}$ is a triple $(\mu(\alpha), \underline{w}, \bar{w}) \in \mathcal{M} \times \mathbb{R} \times \mathbb{R}$ such that, for $k \in \{l, m, h\}$,

$$t^k \in \mu(\alpha) \Rightarrow \begin{cases} V(t^h|\mu(\alpha)) - \bar{w} \geq V(t^m|\mu(\alpha)) - \underline{w} & \text{if } k = h \\ V(t^m|\mu(\alpha)) - \bar{w} \geq V(t^l|\mu(\alpha)) - \underline{w} \text{ and} \\ V(t^m|\mu(\alpha)) - \underline{w} \geq V(t^h|\mu(\alpha)) - \bar{w} & \text{if } k = m \\ V(t^l|\mu(\alpha)) - \underline{w} \geq V(t^m|\mu(\alpha)) - \bar{w} & \text{if } k = l . \end{cases} \quad (6)$$

That is, within any team matched under $\mu(\alpha)$, none of its members expects a higher payoff by forming another team with a worker of a type different from his/her current match. In other words, in an equilibrium there are no incentives for workers to break away from the current pair and form a new pair. This captures the standard notion of stability in the matching literature (see e.g. Gale and Shapley, 1962; Chung, 2000). When $\alpha = 1$, Conditions (6) reduce to

$$V(t^h|\mu(1)) - \bar{w} \geq V(t^m|\mu(1)) - \underline{w} \text{ and} \quad (7)$$

$$V(t^l|\mu(1)) - \underline{w} \geq V(t^m|\mu(1)) - \bar{w}. \quad (8)$$

The next result characterizes equilibria in our economy.

Proposition 1 (Equilibrium)

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$. Then, the following statements hold:

- (i) There is an equilibrium with positive assortative matching (PAM, $\alpha = 1$) if and only if

$$\int_0^1 h(y) dy \geq \int_0^1 h(y) dF^{1,m}(y). \quad (9)$$

¹⁸To simplify notation, we drop the dependence of $V(\cdot|\cdot)$ on e .

¹⁹In our economy, teams' payoffs, and hence, payoffs to team members, are random. Since utility is linear in money, workers are risk-neutral and we can conceive of \bar{w} (\underline{w}) as the payoff that a high-skilled (low-skilled) worker expects to receive.

In this case, the equilibrium wages are

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,h}(y) \quad (10)$$

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,l}(y). \quad (11)$$

(ii) There is an equilibrium with $\alpha \in (0, 1)$ if and only if

$$\int_0^1 h(y) dy = \int_0^1 h(y) dF^{\alpha,m}(y). \quad (12)$$

In this case, the equilibrium wages are

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha,h}(y) \quad (13)$$

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha,l}(y). \quad (14)$$

(iii) There is an equilibrium with negative assortative matching (NAM, $\alpha = 0$) if and only if

$$\int_0^1 h(y) dy \geq \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{0,h}(y) + \frac{1}{2} dF^{0,l}(y) \right). \quad (15)$$

In this case, the equilibrium wages satisfy

$$\bar{w} \in \left[\frac{1}{2} \cdot \int_0^1 h(y) dF^{0,h}(y), \int_0^1 h(y) dF^{0,m}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{0,l}(y) \right] \quad (16)$$

$$\underline{w} = \int_0^1 h(y) dF^{0,m}(y) - \bar{w}. \quad (17)$$

Proof:

See Appendix A.1.

□

Our subsequent analysis revolves around Conditions (9), (12), and (15) and how these are affected by globalization. We pay particular attention to Condition (9), which is necessary and sufficient for an equilibrium with positive assortative matching to exist. Because in an equilibrium with PAM there are only high- and low-skilled teams in equal shares and on average they must earn the expected payoff across all teams in the economy, Condition (9) can be rewritten as

$$\int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{1,l}(y) + dF^{1,h}(y)] \geq \int_0^1 h(y) dF^{1,m}(y).$$

This is a standard supermodularity condition. The key observation is that, in our model, supermodularity refers to expected payoffs, which are determined by non-trivial interactions between the competition mode—as reflected in $dF^{1,k}$ —and the payoff scheme—as captured by $h(y)$. The latter is affected by globalization.²⁰ The supermodularity of a team’s overall skill level (or its expected rank) in team members’ skill levels—as e.g. implied by the O-ring theory of Kremer (1993)—is neither necessary nor sufficient for positive assortative matching to be an equilibrium.

Note the generality of our setup with respect to the equilibrium conditions of interest, namely, Conditions (9), (12), (15). We apply the normalization that higher ranks are better ($h'(y) \geq 0$), and we build on our weak notion of skills in the competition as discussed in Section 3.2. Other than that, we are not imposing any restrictions on how workers’ skills translate into team skills, on how a team’s own skill level and the skill levels of all other teams translate into competition outcomes, and on how competition outcomes translate into payoffs. Yet, our setting allows for strong predictions regarding the impact of our notion of globalization on the concentration of talent, as we show in Section 5.

4.2 Existence

After characterizing equilibria, we want to investigate which conditions on the expected payoffs guarantee that an equilibrium exists. To this end, consider the following condition:

Assumption 4 (Existence of equilibrium)

$V(t^k|\mu(\alpha))$ is continuous in α , for $k \in \{l, m, h\}$.

Imposing Assumption 4 guarantees that the (expected) payoff of each team depends continuously on changes in the workers’ matching. A sufficient, but not necessary, condition for this to happen is that the competition outcome itself does not change abruptly when small changes occur in the way workers match in the economy, i.e., that $f^{\alpha,k}$ is continuous in α . Under this mild assumption, mixed teams’ (expected) payoffs also depend continuously on changes in the workers’ matching, both before and after globalization. As we show in Appendix A.2, an equilibrium always exists in this scenario, in which case it is characterized by Proposition 1.

²⁰These discussions also demonstrate why we focus on rank competitions in the baseline version of our model. With rank competition, the direct effect of globalization—a transformation of the payoff scheme—is separated from matching outcomes. Hence, rank competitions allow discussing our main mechanisms of interest in a transparent way. Yet, our main results readily extend to situations with alternative modes of competitions, as we show in Section 6.2.

Proposition 2

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$. Then, under Assumption 4, an equilibrium exists.

Proof:

See Appendix A.2. □

Two remarks are in order. First, Assumption 4 is sufficient but not necessary for an equilibrium to exist. In our subsequent analysis, we do not impose this assumption, unless we explicitly state it otherwise. Second, even if an equilibrium exists (with or without Assumption 4), it need not be unique, unless further conditions are imposed on expected payoffs (Chade and Eeckhout, 2019). These conditions are not needed for what follows, and we therefore relegate our discussion about uniqueness to Appendix B.1.

5 Globalization

We now analyze how globalization impacts the concentration of talent, before we discuss how this affects equilibrium wages.

5.1 Globalization and the concentration of talent

We say that an economy $e \in \mathcal{E}$ satisfies PAM (satisfies NAM) if there are wages \underline{w} and \bar{w} such that $(\mu(1), \underline{w}, \bar{w})$ ($(\mu(0), \underline{w}, \bar{w})$) is an equilibrium of economy e . From Proposition 1 we know that this is the case if and only if $h(\cdot)$ and \mathcal{F} satisfy Condition (9) (Condition (15)). Because these two conditions, as well as the corresponding conditions for equilibria with $\alpha \in (0, 1)$, are invariant with respect to positive affine transformations of the payoff scheme, we henceforth normalize payoffs under globalization such that

$$\int_0^1 h(y) dy = \int_0^1 g(h(y)) dy. \quad (18)$$

Equation (18) highlights the importance of the redistributive aspects of globalization for our analysis.

The following technical result is central for our understanding of how globalization impacts the concentration of talent in a market. It also allows us to readily extend our main insights to alternative setups, as discussed in Section 6.

Lemma 1

Let $h \in \mathcal{H}$ and $g \in \mathcal{G}$, and consider a continuously differentiable CDF, $F(\cdot)$, with support $[0, 1]$, such that

$$\mathcal{A} := \left\{ y \in [0, 1] : \frac{dF(y)}{dy} \geq 1 \right\}$$

is a convex and compact set. Then,

$$\int_0^1 h(y) dy \geq \int_0^1 h(y) dF(y) \Rightarrow \int_0^1 g(h(y)) dy \geq \int_0^1 g(h(y)) dF(y). \quad (19)$$

Proof:

See Appendix A.3.

□

Lemma 1 implies that whenever a team that is ranked according to $F(\cdot)$ earns less than the average payoff prior to globalization, this will also be the case after globalization.²¹ The main implication of this result in our setup is that globalization promotes the concentration of talent, as the following theorem shows:

Theorem 1

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then,

- (i) if $(\mathcal{W}, h, \mathcal{F})$ satisfies PAM, so does $(\mathcal{W}, g \circ h, \mathcal{F})$;
- (ii) sometimes $(\mathcal{W}, g \circ h, \mathcal{F})$ satisfies PAM and $(\mathcal{W}, h, \mathcal{F})$ does not.

Proof:

See Appendix A.4.

□

The above result states that whenever there is an equilibrium with positive assortative matching prior to globalization, such an equilibrium also exists with globalization. Moreover, in some cases an equilibrium with PAM exists with globalization but not without. The basic intuition is the following: Globalization rewards teams ranked high in the market. When compared to a mixed team, teams whose members are matched positively assorted are more likely to achieve extreme ranks and, in particular, to be ranked at the top. This is because

²¹When set \mathcal{A} contains 0 (1), both (none of the) inequalities in (19) are (is) satisfied, and thus (19) itself follows trivially. Whenever $\{0, 1\} \cap \mathcal{A} = \emptyset$, the inequalities are satisfied for some $h \in \mathcal{H}$ only. Nevertheless, (19) always holds.

the rank distribution of the mixed teams is biased towards achieving mid-range ranks, while low-skilled teams (high-skilled teams) are biased towards achieving low-range ranks (high-range ranks). In other words, teams whose members are matched positively assort will (on average) benefit more from a globalization-induced amplified ‘superstar effect’.

While Theorem 1 analyzes the existence of equilibria with positive assortative matching, the concentration of talent also increases when moving out of an equilibrium with negative assortative matching. It turns out that globalization also promotes the concentration of talent in this latter sense, as shown next.

Theorem 2

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then,

- (i) if $(\mathcal{W}, g \circ h, \mathcal{F})$ satisfies NAM, so does $(\mathcal{W}, h, \mathcal{F})$;
- (ii) sometimes $(\mathcal{W}, h, \mathcal{F})$ satisfies NAM and $(\mathcal{W}, g \circ h, \mathcal{F})$ does not.

Proof:

See Appendix A.5.

□

Clearly, PAM ($\alpha = 1$) and NAM ($\alpha = 0$) represent two extremes of the one-dimensional space $\{\mu(\alpha)\}_{\alpha \in [0,1]}$. Theorems 1 and 2 are therefore *local*, in the sense that they are informative about one value of α only. More generally, one could ask whether, as a result of globalization, the concentration of talent increases by shifting the economy from an equilibrium with $\mu(\alpha)$ to an equilibrium with $\mu(\alpha')$, where $\alpha, \alpha' \in [0, 1]$ satisfy $\alpha \leq \alpha'$. In general, this depends on the (global) behavior of $V(t^m | \mu(\alpha))$ under payoff schemes $h(\cdot)$ and $g(h(\cdot))$. Yet, our setup allows for predictions regarding the equilibria with lowest and highest concentration of talent, respectively, if we impose two conditions: First, a team’s expected payoff depends continuously on the economy-wide matching, as covered by Assumption 4. This is a rather mild assumption and mainly of technical nature—we refer to Section 4.2. Second, a mixed team is more likely to achieve mid-range ranks also in economies where not all other teams are matched positively assort. This is formalized next.

Assumption 1 (continued)

(iii) The set

$$\mathcal{A}_\alpha^\mathcal{F} := \{y \in [0, 1] : f^{\alpha, m}(y) \geq 1\}$$

is convex and compact for all $\alpha \in (0, 1)$.

Assumption 1(iii) is a natural extension of parts (i) and (ii), and its rationale has already been discussed in Section 3.

Now, let $\underline{\alpha}_h$ ($\bar{\alpha}_h$) denote the equilibrium with lowest (highest) concentration of talent prior to globalization and, analogously for $\underline{\alpha}_g$ ($\bar{\alpha}_g$). The latter refers to the case after globalization. With this notation, we obtain the following result regarding the global effect of globalization on the concentration of talent:

Theorem 3

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then, under Assumption 4,

$$(i) \quad \underline{\alpha}_h \leq \underline{\alpha}_g$$

$$(ii) \quad \bar{\alpha}_h \leq \bar{\alpha}_g.$$

Proof:

See Appendix A.6. □

Theorem 3 shows that our setup allows for predictions regarding the effect of globalization on the concentration of talent above and beyond equilibria with positive and negative assortative matching. Specifically, it says that no matter the set of equilibria that exist with and without globalization, it must always be that the equilibrium with highest (lowest) concentration of talent is one with (without) globalization, and this must in particular be true if we focus on *mixed equilibria* beyond PAM and NAM.²²

It is worth noting that Theorem 3 further allows addressing a potential caveat pertaining to Theorem 1. In the latter result, we limit attention to the existence of equilibria with positive assortative matching, without considering uniqueness. Hence, one might wonder whether there might be a unique equilibrium with positive assortative matching prior to globalization, but such an equilibrium might not be unique with globalization. It turns out that under the mild regularity conditions underlying Theorem 3 this cannot occur, as it would contradict Theorem 3(i).

To sum up, Theorems 1 to 3 reveal that our theory predicts strong implications of globalization for the concentration of talent. This has important distributional consequences, as we discuss next.

²²In Theorem 3, $\underline{\alpha}_h, \underline{\alpha}_g, \bar{\alpha}_h, \bar{\alpha}_g \in [0, 1]$. Hence, under Assumption 1 and Assumption 4, Theorem 3 implies the results of Theorems 1 and 2. We stress, however, that Assumption 4 is not needed for Theorems 1 and 2.

5.2 Globalization and wage inequality

Although our main interest is to understand how globalization impacts matching outcomes, it is worth mentioning that a globalization-induced increased concentration of talent has distributional consequences. It is well known that conditional on positive assortative matching, globalization can increase (decrease) *relative* wages of high-skilled (low-skilled) workers via amplified superstar effects.²³ While under plausible restrictions this is also the case here, our setup is general enough to accommodate situations where this direct effect of globalization on wages might not take place. More importantly, there is an additional, indirect effect, since the matching might also change as a result of globalization. To see this, note that we can decompose the overall wage effect for low-skilled workers as follows

$$\begin{aligned} & \frac{1}{2} \cdot \int_0^1 g(h(y)) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_h, l}(y) \\ &= \left[\frac{1}{2} \cdot \int_0^1 g(h(y)) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_g, l}(y) \right] \\ &\quad + \left[\frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_h, l}(y) \right], \end{aligned} \tag{20}$$

and analogously for the wage of the high-skilled workers. In the above equation, we have used α_g (α_h) to denote the matching outcome with (without) globalization, and we have assumed that $\alpha_g \geq \alpha_h > 0$, for simplicity, so that there is no NAM. The first term on the right-hand side of Equation (20) captures the effect of globalization conditional on the matching. For $\alpha_g = 1$ it corresponds to the effect previously considered in the literature. The second term in brackets captures the additional effect that arises from a potential change in the matching, i.e. it is the change in the wage of the low-skilled workers conditional on the payoff scheme.

Now, recall that for $\alpha > 0$ the equilibrium conditions of Proposition 1 refer to the payoff distribution of mixed teams, but do not directly refer to the payoff distributions of high-skilled teams or low-skilled teams. The latter distributions nonetheless determine the wages. This means that the sign of both terms on the right-hand side of Equation (20) is ambiguous, unless more structure is imposed. Again, this is due to the generality of our setup. Yet, plausible restrictions allow for stronger comparative statics. An interesting case in this regard is one where $F^{\alpha_h, l}(y) \leq F^{\alpha_g, l}(y)$ for all $y \in [0, 1]$, i.e. $F^{\alpha_h, l}(y)$ first-order stochastically dominates distribution $F^{\alpha_g, l}(y)$ (Mas-Colell et al., 1995, Prop. 6.D.1). In this case, the second term on the right-hand side of Equation (20) is negative. This suggests that in a

²³Cf. the literature review in Section 2.

world where low-skilled teams are relatively better at competing against mixed teams than against assortatively matched teams, it is more natural to expect that the net effect of globalization on relative wages of the low-skilled workers is negative above and beyond any potential effect conditional on the matching.²⁴

6 Extensions and Alternative Specifications

In this section, we discuss several extensions of our baseline setup and show that our main result—Theorem 1—readily extends to these alternative settings.

6.1 Several types and team members

We start by analyzing how Theorem 1 can be extended to the case with several skill types and several team members. Suppose there are $S \geq 2$ types with arbitrary population shares, which can be identified by their skill level $s \in \mathcal{S}$, where \mathcal{S} denotes the set of skills available in the economy. Let $N \geq 2$ denote the number of workers in each team t , with $\mathcal{S}_t = \{s_t^1, s_t^2, \dots, s_t^N\}$ denoting the multiset of skill levels in team t . Then, there is an equilibrium with positive assortative matching if and only if for every worker type $s \in \mathcal{S}$,

$$V(t^{\{s\}^N} | PAM) - (N-1) \cdot w^s \geq \max_{\hat{\mathcal{S}} \in \mathcal{S}^{N-1}} \left\{ V(t^{s \cup \hat{\mathcal{S}}} | PAM) - \sum_{\hat{s} \in \hat{\mathcal{S}}} w^{\hat{s}} \right\}.$$

Here, w^s denotes the equilibrium wage for a worker with skill level s , $t^{\mathcal{S}_t}$ identifies a team with workers of skill levels \mathcal{S}_t , and

$$V(t^{\mathcal{S}_t} | PAM) = \int_0^1 h(y) dF^{PAM, \mathcal{S}_t}(y)$$

indicates the expected payoff of team $t^{\mathcal{S}_t}$, assuming that all other workers are matched positively assorted. As in the previous sections, $F^{PAM, \mathcal{S}_t}(y)$ is the rank distribution of a team t with skill levels \mathcal{S}_t when competing against teams that are arranged according to PAM, and $f^{PAM, \mathcal{S}_t}(y) := \frac{dF^{PAM, \mathcal{S}_t}}{dy}(y)$ is the associated PDF.²⁵ It can be verified that wages in an equilibrium with positive assortative matching satisfy

$$w^s = \frac{1}{N} \cdot V(t^{\{s\}^N} | PAM), \text{ for all } s \in \mathcal{S}.$$

²⁴An increased concentration of talent may well have distributional consequences above and beyond any immediate wage effect, e.g. in the presence of knowledge spillovers. A thorough investigation of such effects is beyond the scope of our paper and is left for future research.

²⁵We assume that for all $\mathcal{S}_t \in \mathcal{S}^N$, f^{PAM, \mathcal{S}_t} satisfies all the regularity conditions imposed on $f^{\alpha, k}$ in Section 3.

Using the above equilibrium wages and rearranging terms, we obtain that there is an equilibrium with positive assortative matching if and only if

$$\frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} V(t^{\{s\}^N} | PAM) \geq V(t^{\mathcal{S}_t} | PAM) , \text{ for all } \mathcal{S}_t \in \mathcal{S}^N. \quad (21)$$

This is again a supermodularity condition.

Now, consider the following adaptation of Assumption 1(i):

Assumption 1'

For any multiset of skills $\mathcal{S}_t \in \mathcal{S}^N$, it holds that

$$\mathcal{A} := \left\{ y \in [0, 1] : f^{PAM, \mathcal{S}_t}(y) \geq \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) \right\} \quad (22)$$

is a convex and compact set.

Analogously to the case of two types and two team members, the most natural interpretation of Condition (22) is that the rank distributions of mixed teams are biased towards achieving mid-range ranks when compared to assortatively matched teams of the types corresponding to its team members. For a given $\mathcal{S}_t \in \mathcal{S}^N$, we can rewrite Condition (21) as

$$\int_0^1 h(y) \cdot \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) dy \geq \int_0^1 h(y) \cdot f^{PAM, \mathcal{S}_t}(y) dy.$$

Letting $F(y) := \int_0^y \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(x) dx$ and using the change of variables $z := F(y)$ yields

$$\int_0^1 h(F^{-1}(z)) dz \geq \int_0^1 h(F^{-1}(z)) \cdot \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} dz,$$

where $f(y) := \frac{d(F(y))}{dy}$.²⁶ Clearly, $\frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))}$ is a PDF on $[0, 1]$,²⁷ and

$$\mathcal{A} := \left\{ z \in [0, 1] : \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} \geq 1 \right\}$$

is a convex and compact set by Assumption 1'. Hence, we can apply Lemma 1 to generalize Theorem 1 to the case with many types and team members.

²⁶We have simplified the exposition by assuming that $f(\cdot)$ has continuous support. At the expense of additional notational complexity, this simplification can easily be dispensed with.

²⁷In particular, $\frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))}$ is positive and satisfies

$$\int_0^1 \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} dz = \int_0^1 f^{PAM, \mathcal{S}_t}(y) dy = 1.$$

6.2 Beyond rank competition

Thus far we have considered economies with competition for rank. In such cases, there is a clear distinction between rank-dependent payoffs that are affected by globalization, on the one hand, and the competition and matching outcomes that impact teams' rank distributions, on the other. The focus on rank competitions therefore allows us to discuss our main mechanisms of interest in a transparent way. Yet, our main results also apply to models where a team's payoff directly depends on its own productivity and the productivities of all other teams in the economy, as we show next. To simplify the exposition, we return to considering two types of workers only—high-skilled and low-skilled—that form teams of two, but arguments along the lines of Section 6.1 imply that the results shown extend to scenarios with more types and team members.

Let us assume that each team forms and then receives a random productivity draw φ from a publicly-known, skill-dependent probability distribution $B^k(\varphi)$, $k \in \{l, m, h\}$, that has support $\Phi \subseteq \mathbb{R}_+$, with density function $b^k(\varphi) := \frac{dB^k(\varphi)}{d\varphi}$. After receiving their productivity draw, teams compete in a market where each team's payoff depends on its own productivity and the productivities of all other teams in the economy, analogously to e.g. a Melitz (2003)-model (see Section 7.2.2 for a discussion). These productivities depend on the (whole) matching. With a continuum of workers, however, there is no aggregate uncertainty about the distribution of productivities in the economy, once the matching is given. Hence, we can summarize the entire distribution of productivities in the economy by a parameter α . As in Section 3, it denotes the share of teams whose members are matched positively assort. Accordingly, we use $\pi^\alpha(\varphi)$ to denote the payoff of a team with productivity φ given matching α . Then, the expected payoff of a team $k \in \{l, m, h\}$ given PAM is

$$V(t^k | \mu(1)) = \int_{\varphi \in \Phi} \pi^1(\varphi) dB^k(\varphi),$$

and PAM is an equilibrium if and only if

$$\int_{\varphi \in \Phi} \pi^1(\varphi) dB^m(\varphi) \leq \int_{\varphi \in \Phi} \pi^1(\varphi) \cdot \frac{1}{2} \cdot [dB^l(\varphi) + dB^h(\varphi)].$$

Now, consider the following adaptation of Assumption 1:

Assumption 1''

The set

$$\mathcal{A} := \left\{ \varphi \in \Phi : b^m(\varphi) \geq \frac{1}{2} [b^l(\varphi) + b^h(\varphi)] \right\} \quad (23)$$

is convex and compact.

Then—as we show in Appendix B.2—we can again use Lemma 1 to show that Theorem 1 generalizes to this case for any form of globalization, $g(\cdot)$, that is an increasing, convex transformation of payoffs conditional on PAM, $\pi^1(\varphi)$.

6.3 Migration

We conclude this section with a brief discussion of migration. In our analysis, we have taken the pool of workers as given and, in particular, we have assumed that it is itself not affected by globalization. While in many instances this may be a reasonable approximation given that capital, goods, and services tend to be more mobile than people, in other cases—such as our illustrative example from European football—workers tend to be more mobile. It is thus relevant to discuss how our results are affected by labor mobility. To that end, we consider two polar cases.

First, assume that there are two perfectly symmetric economies that were initially separated and then integrated their labor markets, but which still have separate competitions. In the case of European football, for example, teams still compete in their national leagues. Our results remain valid in this first polar approach to labor mobility. That is, globalization in the form of a convex transformation of payoffs increases the concentration of talent with this extreme form of labor mobility as well. This is because with perfectly mobile labor, wages for high- and low-skilled workers have to be the same across countries. Hence, migration will not change the skill composition of the two economies. Referring to our football example, indeed it is not the case that the very best players all play in one country.

Our second polar case regarding labor mobility assumes that the two countries will integrate completely, i.e. both their competition and their labor markets will be merged. This corresponds to a simple scaling of our economy, and our results directly apply as long as the competition and the payoff structure are scale invariant.²⁸

7 Micro-foundations

In the previous sections, we have presented a simple reduced-form analysis of how globalization impacts the concentration of talent across competing teams. In this section, we present

²⁸In this case, integrating the economies yields a payoff scheme $\tilde{h}(y)$, with $\tilde{h}(2y) := h(y)$, and a rank distribution for a team $k \in \{l, m, h\}$ in an economy where a share α of teams have members who are matched positively assorting $\tilde{f}^{\alpha,k}(y)$, where $\tilde{f}^{\alpha,k}(2y) := f^{\alpha,k}(y)$. A simple change of variables $x := 2y$ then implies that integration will not impact equilibrium outcomes.

micro-foundations for our two main assumptions: Assumption 1 (about competition) and Assumption 3 (about globalization). We begin with the former, which specifically refers to how the competition mode determines the ranking, and then consider the latter, which specifically refers to how globalization impacts payoffs.

7.1 Micro-foundations for the competition mode

7.1.1 Head-to-head competition

Suppose that all teams compete head to head and are then ranked according to the number of victories. This is in line with competition in a sports league, for example. More specifically, assume that each time two teams meet, there is a fixed probability $p > \frac{1}{2}$ that the higher skilled team wins, with that probability being $\frac{1}{2}$ if two equally skilled teams compete. Then, as the number of games that a particular team is involved in goes to infinity, the ratio of victories for this team will converge to its expected value by the law of large numbers.²⁹ As a consequence, the rank distribution of a low-skilled team will be uniform on $[0, \frac{\alpha}{2}]$, that of a mixed team will be uniform on $[\frac{\alpha}{2}, 1 - \frac{\alpha}{2}]$ and that of a high-skilled team uniform on $[1 - \frac{\alpha}{2}, 1]$.³⁰ This implies that the set \mathcal{A}_α^F is (asymptotically) convex and compact, in which case Assumption 1 is satisfied.

7.1.2 Patent race

Assumption 1 also arises naturally when teams compete against each other in a patent or innovation race. To show this, assume that after teams have formed, they are ranked according to the timing of first events drawn from a Poisson process with skill-dependent arrival rate $\lambda^k > 0$, with $k \in \{l, m, h\}$. That is, the cumulative distribution function for the time of invention of a team with skill level λ^k is³¹

$$B^k(z) = 1 - e^{-\lambda^k z}.$$

²⁹We follow the convention in the economics literature and apply the law of large numbers to a continuum of random variables.

³⁰To see this note that the expected share of victories for a low-skilled team will be $p^l = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot (1 - p)$, that of a mixed team will be $p^m = \frac{\alpha}{2} \cdot p + \frac{\alpha}{2} \cdot (1 - p) + \frac{1-\alpha}{2} = \frac{1}{2}$, and that of a high-skilled team will be $p^h = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot p$. It is then straightforward to verify that for every $p > \frac{1}{2}$, we must have $p^l < p^m < p^h$. The resulting rank CDFs are not continuously differentiable. Note, however, that we could easily smoothen things by assuming that the overall skill level of a team is itself subject to a random shock such that some teams of low-skilled workers, for example, end up being relatively high skilled.

³¹See Loury (1979) and Dasgupta and Stiglitz (1980) for seminal contributions using Poisson processes in the modeling of patent races.

As usual, we let $b^k(z)$ denote the corresponding PDF. In an economy with a share α of all teams whose members are matched positively assortated, the (expected) rank $y \in [0, 1]$ of a team that invents at some time $z \in \mathbb{R}_+$ is then given by

$$y = r^\alpha(z) = \frac{\alpha}{2}e^{-\lambda^l z} + \frac{\alpha}{2}e^{-\lambda^h z} + (1 - \alpha)e^{-\lambda^m z},$$

where $(\alpha/2)e^{-\lambda^l z}$ is the measure of low-skilled teams that have not yet innovated at time z and analogously for $(\alpha/2)e^{-\lambda^h z}$ and $(1 - \alpha)e^{-\lambda^m z}$. As we show in Appendix B.3, the rank distribution of the mixed team satisfies Assumption 1 for any $\lambda^l, \lambda^m, \lambda^h > 0$. Of course, if skills are to be meaningful, it should be $\lambda^l \leq \lambda^m \leq \lambda^h$.

7.1.3 Pareto distribution of team's productivity

Finally, we show that Assumption 1'' is satisfied if a team's productivity is drawn from a Pareto distribution with skill-dependent location parameter. More specifically, given $\gamma > 0$ let

$$b^k(\varphi) := \begin{cases} \frac{\gamma \varphi^{k\gamma}}{\varphi^{\gamma+1}} & \text{if } \varphi \geq \varphi^k \\ 0 & \text{otherwise} \end{cases}$$

be the PDF for a team with skill level $k \in \{l, m, h\}$, and then suppose that $\underline{\varphi}^l \leq \underline{\varphi}^m \leq \underline{\varphi}^h$. It immediately follows that the set

$$\mathcal{A} := \left\{ \varphi \in \Phi : b^m(\varphi) \geq \frac{1}{2} [b^l(\varphi) + b^h(\varphi)] \right\}$$

is convex. Hence, Assumption 1'' is satisfied.³²

7.2 Micro-foundations for globalization as convex transformation

To rationalize Assumption 3, we build on the models of Rosen (1981) and Melitz (2003), respectively.

³²If $\underline{\varphi}^{m\gamma} \geq \frac{1}{2} [\underline{\varphi}^{l\gamma} + \underline{\varphi}^{h\gamma}]$, we have $\mathcal{A} = [\underline{\varphi}^m, \infty)$. If $\underline{\varphi}^{m\gamma} < \frac{1}{2} [\underline{\varphi}^{l\gamma} + \underline{\varphi}^{h\gamma}]$, we have $\mathcal{A} = [\underline{\varphi}^m, \underline{\varphi}^h]$. These sets are not compact. Nevertheless, the former is compact when considering a truncated Pareto distribution with truncation threshold $\bar{\varphi}$, and we can approximate the Pareto distribution by choosing $\bar{\varphi}$ large. The latter is compact when considering the following distribution for the high-skilled team

$$b^h(\varphi) = \frac{1}{\epsilon} \cdot \int_{-\epsilon/2}^{\epsilon/2} b_\delta^h(\varphi) d\delta,$$

where $b_\delta^h(\varphi)$ denotes the Pareto distribution with lower-bound $\underline{\varphi}_\delta^h := \underline{\varphi}^h + \delta$. Again, by choosing $\epsilon \rightarrow 0$ we can approximate the Pareto distribution.

7.2.1 Rosen (1981) with competing teams

In his seminal paper, Rosen (1981) shows how small differences in talent can result in large differences in sales and income at the top in the case of markets with imperfect substitutability of quantity for quality. Moreover, he shows that globalization—namely, an increase in the size of the market—results in a convex transformation of payoffs. In this section, we briefly discuss a simple variant of his model with competing teams and show how it maps into our reduced-form analysis.

Suppose that consumers demand overall services $x := nz$ from competing teams, where z is the quality of service provided by a team and n is the quantity consumed, such as the number of games attended. Consumers face a fixed cost s per unit of service consumed, which we can think of as representing e.g. the time cost associated with consuming the service. Hence, the total cost of consuming n units of a service with quality z is equal to $n(p(z) + s)$, where $p(z)$ is the price per unit of service of quality z . The cost per effective unit is $v := (p(z) + s)/z$. The latter cost is constant across teams in equilibrium. Rosen (1981) allows for the existence of internal and external dis-economies of scale, i.e. the cost of providing m units of service, $C(m)$, is increasing and convex. In addition, the quality of service $z = z(y, m)$ is decreasing in the number of units sold, reflecting congestion effects.³³ In this setup, y is a measure of the underlying value of the service provided by a team, and we can allow for different interpretations of y . In what follows, we think of y simply as the rank of the team—reflecting the fact that supporters enjoy seeing their team winning—but it could also express the skill level of the team drawn from a random distribution as discussed in Section 6.2. Rosen (1981) shows that the payoff of a team ranked y , $h(y)$, satisfies

$$h''(y) = v(z_y + mz_{ym}) \frac{\partial m}{\partial y} + vmz_{yy},$$

i.e. the revenue function is convex over ranks if $z_{yy} \geq 0$ and $z_{ym} \geq 0$, where the latter inequality implies that higher ranked teams are better at serving larger audiences. The intuition is that in this case higher-ranked teams cannot only charge a higher price, but it is also profitable for them to serve larger audiences, which implies a convex payoff-scheme. The important point to note is that convexity increases in v , which is the market price per unit of service. That is, when in the wave of globalization demand for these services goes up so that v increases, the payoff scheme becomes more convex, i.e. Assumption 3 holds. Hence, our reduced-form analysis applies to this variant of the Rosen (1981) model, provided that the competition satisfies Assumption 1.

³³For example, Rosen (1981) suggests that it is more valuable to attend a concert in a small concert hall than in the Yankee Stadium.

7.2.2 Fixed cost of market entry: Melitz (2003)-model with entrepreneurial teams

Assumption 3 is also naturally satisfied if teams have the opportunity to access foreign markets, albeit at a fixed cost. Suppose that the gains from entering a foreign market are increasing with a team's rank in its domestic economy. In the case of European football, we may think of entering a foreign market as actively trying to increase a team's fan base in these markets to raise revenues via sponsoring, merchandising, or licensing, for example. Such endeavors are naturally more promising for teams that perform well in their domestic leagues.³⁴ Now, assume for simplicity that payoffs generated abroad, $\tilde{h}(y)$, are proportional to the domestic payoffs $h(y)$, i.e.

$$\tilde{h}(y) = \lambda \cdot h(y)$$

for some constant $\lambda > 0$. Teams will enter the foreign market only if this is profitable, implying that the total payoff of a team ranked y is

$$g(h(y)) = h(y) + \max\{0, \lambda \cdot h(y) - c\} .$$

It is straightforward to verify that $g(\cdot)$ is increasing and convex.^{35,36}

The same logic also implies that our analysis directly applies to a simple variant of a Melitz (2003)-model with entrepreneurial teams. In this variant, agents are either high-skilled or low-skilled. High-skilled agents are better entrepreneurs, but they have no advantage when employed as a worker. There is an initial stage where agents decide whether or not to become an entrepreneur. Entrepreneurs match to form entrepreneurial teams of two and found a firm. As in the workhorse version of a Melitz (2003)-model, each firm is equipped with a distinct variety, and receives a random productivity draw, φ , from a known Pareto distribution. In this variant, however, the minimum value of φ is increasing in the skill

³⁴In Germany, for example, Bayern Munich and Borussia Dortmund, the biggest and most successful football clubs in recent years, are most actively promoting their teams abroad and they are the only clubs running foreign offices (see <https://www.welt.de/sport/article157261763/Das-Millionenspiel-der-Bundesligaklubs-in-Uebersee.html>). They also have by far the most facebook likes outside of Germany (see <http://meedia.de/2015/09/23/bundesliga-bis-3-liga-das-grosse-facebook-ranking-der-fussballclubs/>, retrieved on 26 October 2017).

³⁵In this example $g(h(\cdot))$ is not differentiable for all $y \in [0, 1]$. Nevertheless, our results do not hinge crucially on this regularity assumption, which we have only imposed for simplicity.

³⁶We consider the case where globalization gives rise to a convex transformation of teams' payoffs directly. Alternatively, globalization may yield a convex transformation of firm sizes and then spill over to the compensation of managerial teams. Gabaix and Landier (2008), for example, present a model where at the top CEO pay (executive board pay in a simple variant with managerial teams) is proportionate to a power function of firm size, i.e. for all power coefficients larger than or equal to one, a globalization-induced convex transformation of firm sizes with $g(0) = 0$ translates into a convex transformation of executive compensation.

level of the entrepreneurial team. As in the canonical Melitz (2003)-model, firms with productivity draws above some endogenous threshold level will start operating, while all other firms will exit immediately. This implies that under autarky, a firm’s profit is a piecewise linear and convex function of $\varphi^{\sigma-1}$, where $\sigma > 1$ is the constant elasticity of substitution between varieties. The most interesting case is one with selection into exporting, in line with empirical facts. Globalization—a move from autarky to an equilibrium with trade—then implies that the minimum-productivity threshold for firms increases, i.e. the lowest productive firms are forced to exit after a trade liberalization. Firms with intermediate levels of productivity only serve their domestic market, and the most productive firms also export. The key point is that globalization gives rise to a piecewise linear and convex transformation of a firm’s profits as a function of $\varphi^{\sigma-1}$ —we refer to Melitz and Redding (2014) for further details. Moreover, $\varphi^{\sigma-1}$ is Pareto distributed as well, and mixed teams are thus relatively more likely to have mid-range values of $\varphi^{\sigma-1}$, as shown in Section 7.1.3. This implies that this variant of the Melitz (2003)-model reduces to the model analyzed in Section 6.2. We refer to Appendix B.4 for further discussions.³⁷

8 Conclusion

We have investigated the effect of a convex transformation of payoffs on the concentration of talent in large matching markets. We have argued that globalization can attest for this type of transformation of the payoff scheme. This implies that in relative terms, high ranks are rewarded higher prizes *after* globalization than *before*. We have chosen a reduced-form approach to modelling competition that rests on minimal assumptions pertaining to the relationship between skills and market outcomes, and we expect the mechanisms we have considered to be relevant for many actual markets with competing teams. Because payoffs in such markets might be—and often are—influenced by exogenous elements (namely, globalization), our research question clearly seems relevant both theoretically and empirically.

Our main insight is that globalization promotes the concentration of talent, i.e., it may result in the emergence of positive assortative matching. This has important distributional consequences, as it feeds back into the income distribution of modern societies. Potentially

³⁷In the Melitz (2003)-model, the total mass of entering firms is endogenous, which impacts the distribution of skills over entrepreneurial teams. In turn, this mass of entering firms depends on the matching. With a Pareto distribution of firm productivities, however, it does not directly depend on the trade environment. Hence, the endogeneity of the mass of entering firms does not impede the applicability of our main results, as these depend only on the redistributive effects of globalization *conditional* on the matching outcome. See Appendix B.4 for further details.

adverse effects of globalization above and beyond a direct effect on income inequality have received growing attention in recent years (Autor et al., 2014; Che et al., 2016; McManus and Schaur, 2016; Pierce and Schott, 2016). An increased concentration of talent might be an important factor in this regard, because in the presence of learning externalities it may harm low-skilled workers and perpetuate—or even increase—skill differences. More generally, a greater concentration of talent contributes to social segregation. Future work may set out to study such effects and their welfare implications in more detail.

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Appendix

A Proofs

In this Appendix, we prove Propositions 1 and 2, Lemma 1, and Theorems 1–3.

A.1 Proof of Proposition 1

In the following, we show the three parts of Proposition 1.

Proof of Part (i): In an equilibrium with positive assortative matching, $(\mu(1), \underline{w}, \bar{w})$, Conditions (6) reduce to Conditions (7) and (8). Because there is a continuum of workers—and hence every worker can always find another worker with whom to match—, it follows that

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,h}(y)$$

and

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,l}(y),$$

which yields Equations (10) and (11). With these expressions for wages, both Conditions (7) and (8) then reduce to the same condition, namely

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) \geq V(t^m|\mu(1)). \quad (\text{A.1})$$

Accordingly, there is an equilibrium with positive assortative matching if and only if (A.1) is satisfied. Now, note that for $\alpha = 1$, Equation (1) reduces to

$$1 = \frac{1}{2} \cdot \frac{dF^{1,l}(y)}{dy} + \frac{1}{2} \cdot \frac{dF^{1,h}(y)}{dy}.$$

Using the above equation and Equation (5), we can write

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) = \int_0^1 h(y) \cdot \left(\frac{1}{2} \cdot dF^{1,l}(y) + \frac{1}{2} \cdot dF^{1,h}(y) \right) = \int_0^1 h(y) dy,$$

and

$$V(t^m|\mu(1)) = \int_0^1 h(y) \cdot dF^{1,m}(y).$$

Finally, using the above two equations in (A.1), we obtain Condition (9). This concludes the proof of Part (i).

Proof of Part (ii): First of all, note that the same argument as in Part (i) implies that in an equilibrium $(\mu(\alpha), \underline{w}, \bar{w})$ with $\alpha \in (0, 1)$, wages are given by (13) and (14). Further, with $\alpha \in (0, 1)$, Conditions (6) reduce to

$$\begin{aligned} V(t^h|\mu(\alpha)) - \bar{w} &= V(t^m|\mu(\alpha)) - \underline{w}, \\ V(t^l|\mu(\alpha)) - \underline{w} &= V(t^m|\mu(\alpha)) - \bar{w}. \end{aligned}$$

Using the equilibrium wages given by Equations (13) and (14) as well as Equation (5), the above two equations can both be rearranged to

$$\frac{1}{2} \cdot V(t^h|\mu(\alpha)) + \frac{1}{2} \cdot V(t^l|\mu(\alpha)) = V(t^m|\mu(\alpha)).$$

Finally, Equation (12) is easily obtained if we use Equation (5) again in combination with Equation (1). This finishes the proof of Part (ii).

Proof of Part (iii): With negative assortative matching, Conditions (6) reduce to

$$V(t^m|\mu(0)) - \bar{w} \geq V(t^l|\mu(0)) - \underline{w}, \quad (\text{A.2})$$

$$V(t^m|\mu(0)) - \underline{w} \geq V(t^h|\mu(0)) - \bar{w}. \quad (\text{A.3})$$

On the one hand, if there is indeed an equilibrium $(\mu(0), \underline{w}, \bar{w})$, it must be that

$$\underline{w} = V(t^m|\mu(0)) - \bar{w}. \quad (\text{A.4})$$

The reason is that wages of both members of a mixed team must add up to total expected payoffs of the team. Using Equation (5) on Equation (A.4), we obtain Equation (17), which is necessary for $(\mu(0), \underline{w}, \bar{w})$ to be an equilibrium. On the other hand, adding Conditions (A.2) and (A.3) yields the following further necessary condition

$$\frac{1}{2} \cdot V(t^h|\mu(0)) + \frac{1}{2} \cdot V(t^l|\mu(0)) \leq V(t^m|\mu(0)). \quad (\text{A.5})$$

Condition (15) follows from using Equations (1) and (5) on (A.5). Finally, Condition (16) follows from noting that (i) if $\bar{w} < \frac{1}{2}V(t^h|\mu(0))$, high-skilled workers would benefit from breaking apart from a mixed team and matching another high-skilled worker, (ii) if $\underline{w} < \frac{1}{2}V(t^l|\mu(0))$, low-skilled workers would benefit from breaking apart from a mixed team and then matching another low-skilled worker. Conversely, for any wages \underline{w} and \bar{w} that satisfy these two latter constraints plus Equation (A.4), no worker can profitably deviate from his current match, implying that the negative assortative matching $(\mu(0), \underline{w}, \bar{w})$ is indeed an equilibrium.³⁸ This finishes the proof of Part (iii).

□

³⁸Note that the set defined in (16) is non-empty by Equation (15).

A.2 Proof of Proposition 2

Observe, on the one hand, that for no equilibrium with PAM to exist, we must have

$$\begin{aligned} \frac{1}{2} \cdot V(t^l|\mu(1)) + \frac{1}{2} \cdot V(t^h|\mu(1)) &= \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{1,h}(y) + \frac{1}{2} dF^{1,l}(y) \right) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) dF^{1,m}(y) = V(t^m|\mu(1)), \end{aligned}$$

where the second equality follows from Equation (1). On the other hand, for no equilibrium with NAM to exist, we must have

$$\begin{aligned} V(t^m|\mu(0)) &= \int_0^1 h(y) dF^{0,m}(y) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{0,h}(y) + \frac{1}{2} dF^{0,l}(y) \right) = \frac{1}{2} \cdot V(t^l|\mu(0)) + \frac{1}{2} \cdot V(t^h|\mu(0)). \end{aligned}$$

Now, suppose that the above inequalities are both satisfied. Then, due to Assumption 4, there must exist $\alpha^* \in (0, 1)$ such that

$$V(t^m|\mu(\alpha^*)) = \frac{1}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{1}{2} \cdot V(t^h|\mu(\alpha^*)).$$

Using Equation (1) as well as the above equation, we obtain

$$\begin{aligned} \int_0^1 h(y) dy &= \frac{\alpha^*}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{\alpha^*}{2} \cdot V(t^h|\mu(\alpha^*)) + (1 - \alpha^*) \cdot V(t^m|\mu(\alpha^*)) = V(t^m|\mu(\alpha^*)) \\ &= \int_0^1 h(y) dF^{\alpha^*,m}(y), \end{aligned}$$

and, hence, there is an equilibrium where workers are arranged according to $\mu(\alpha^*)$. This proves our claim that an equilibrium always exists under Assumption 4.

□

A.3 Proof of Lemma 1

It is convenient to denote the CDF corresponding to a uniform random variable on $[0, 1]$ as

$$I(y) = y, \text{ for all } y \in [0, 1],$$

with $i(y) = 1$ (for all $y \in [0, 1]$) denoting the corresponding PDF. Then, let $F(\cdot)$ be a CDF with support on $[0, 1]$, with $f(\cdot)$ denoting the corresponding PDF, such that

$$\mathcal{A} := \{y \in [0, 1] : f(y) \geq 1\} \tag{A.6}$$

is a convex and compact set, which we can therefore write as $\mathcal{A} = [\underline{a}, \bar{a}]$. We recall that

$$F(t) = \int_0^t f(y)dy. \quad (\text{A.7})$$

We distinguish three cases, for all of which we assume $h \in \mathcal{H}$. We also assume that \mathcal{A} and $[0, 1]$ differ on a set of positive measure, since otherwise we must have $F = I$ and the lemma holds trivially.

Case I: $\underline{a} = 0$ (and $\bar{a} < 1$)

In this case, $I(y)$ first order stochastically dominates $F(y)$. Mas-Colell et al. (1995, Proposition 6.D.1) then implies that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)f(y)dy$$

for all $h \in \mathcal{H}$ (and in particular for $g \circ h$). This means that in this case, the inequalities in Lemma 1 are always satisfied (with or without globalization).

Case II: $0 < \underline{a} < \bar{a} < 1$

From (A.6) and (A.7) and the fact that $F(0) = I(0) = 0$ and $F(1) = I(1) = 1$, it follows that there is $b^* \in (\underline{a}, \bar{a})$ such that

$$F(y) \begin{cases} \leq I(y) = y & \text{if } y \leq b^* \\ = I(y) = y & \text{if } y = b^* \\ \geq I(y) = y & \text{if } y \geq b^* \end{cases}.$$

That is, $F(\cdot)$ crosses $I(\cdot)$ once and from below. In other words, $(I - F)$ has a single change from positive to negative. Moreover, recall that we are assuming $g'(y) \geq g'(0) > 0$ for all $y \in [0, 1]$. This implies that the inverse $g^{-1}(\cdot)$ of $g(\cdot)$ is defined for all $y \in [0, 1]$, and we have

$$\frac{d}{dy}g^{-1}(y) = \frac{1}{g'(y)} > 0,$$

and

$$\frac{d^2}{dy^2}g^{-1}(y) = -\frac{1}{(g'(y))^2} \cdot g''(y) \leq 0.$$

That is, h can be obtained from $g \circ h$ through a concave transformation. From Proposition 6.C.2. in Mas-Colell et al. (1995), we obtain that

$$r_h(y) := -\frac{h''(y)}{h'(y)} \geq -\frac{(g \circ h)''(y)}{(g \circ h)'(y)} := r_{g \circ h}(y).$$

Then, Theorem 3 in Hammond, John S. III. (1974) implies that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)f(y)dy \implies \int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))f(y)dy. \quad (\text{A.8})$$

Case III: $\bar{a} = 1$ (and $\underline{a} > 0$)

In this case,³⁹

$$\begin{aligned}\int_0^1 h(y) \cdot [f(y) - 1] dy &= \int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy + \int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy \\ &> h(\underline{a}) \cdot \left[\int_0^{\underline{a}} [f(y) - 1] dy + \int_{\underline{a}}^1 [f(y) - 1] dy \right] = 0.\end{aligned}\quad (\text{A.9})$$

The last equality is a consequence of $f(\cdot)$ being a PDF with domain $[0, 1]$. To show that the inequality holds, we note that $h(\cdot)$ is increasing and distinguish two cases. First, let

$$h(\underline{a}) > 0.$$

Then, the inequality in (A.9) holds because

$$\int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy > h(\underline{a}) \cdot \int_0^{\underline{a}} [f(y) - 1] dy$$

and

$$\int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy \geq h(\underline{a}) \cdot \int_{\underline{a}}^1 [f(y) - 1] dy.$$

Second, let

$$h(\underline{a}) = 0.$$

Then, the inequality in (A.9) holds because

$$\int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy = h(\underline{a}) \cdot \int_0^{\underline{a}} [f(y) - 1] dy = 0$$

and

$$\int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy > h(\underline{a}) \cdot \int_{\underline{a}}^1 [f(y) - 1] dy = 0.$$

That is, we have shown that in this case we always have

$$\int_0^1 h(y) dy < \int_0^1 h(y) f(y) dy,$$

i.e., the statement of the lemma trivially holds because the inequalities are never satisfied (with or without globalization).

□

³⁹In this case, $F(y)$ first order stochastically dominates $I(y)$. Nevertheless, we cannot directly apply Mas-Colell et al. (1995, Proposition 6.D.1) because we need the inequality in (A.9) to be strict. Theorem 3 in Hammond, John S. III. (1974) could be used for Case III, although no proof is provided in his paper.

A.4 Proof of Theorem 1

Part (i) follows immediately from Lemma 1, which implies that whenever $h(y)$ and $F^{1,m}(y)$ satisfy Condition (9), so will $g(h(y))$ and $F^{1,m}(y)$. We prove part (ii) by means of an example with the desired property. That is, we want to find $h \in \mathcal{H}$, $g \in \mathcal{G}$ and $F^{1,m}(\cdot)$ such that

$$\int_0^1 h(y) \cdot dF^{1,m}(y) > \int_0^1 h(y) dy = \int_0^1 g(h(y)) dy \geq \int_0^1 g(h(y)) \cdot dF^{1,m}(y).$$

Accordingly, let $h(x) = x^{1/2}$ and $g(x) = \frac{4(1-\delta)}{3} \cdot x^2 + \delta \cdot x$, for $\delta > 0$. Clearly, $h \in \mathcal{H}$ and $g \in \mathcal{G}$. It is then a matter of simple algebra to verify that

$$\int_0^1 h(y) dy = \int_0^1 g(h(y)) dy = \frac{2}{3}.$$

Next, consider $F^{1,m}(\cdot)$ defined for every $\varepsilon > 0$ as follows

$$dF^{1,m}(y) = \begin{cases} 1/\varepsilon & \text{if } 0.47 - \varepsilon/2 \leq y \leq 0.47 + \varepsilon/2 \\ 0 & \text{otherwise} \end{cases}.$$

For $\varepsilon > 0$ and $\delta > 0$ arbitrarily low, we obtain

$$\int_0^1 g(h(y)) \cdot dF^{1,m}(y) \approx 0.63 < \frac{2}{3}$$

and

$$\int_0^1 h(y) \cdot dF^{1,m}(y) \approx 0.69 > \frac{2}{3}.$$

Accordingly, $(\mathcal{W}, g \circ h, \bar{\mathcal{F}})$ satisfies PAM, but $(\mathcal{W}, h, \bar{\mathcal{F}})$ does not. □

A.5 Proof of Theorem 2

The proof of Theorem 2 closely follows the arguments in the proofs of Lemma 1 and Theorem 1 and can be provided upon request. Here, we just show Part (ii) by means of an example with the desired property. That is, we want to find $h \in \mathcal{H}$, $g \in \mathcal{G}$, $F^{0,l}(\cdot)$ and $F^{0,h}(\cdot)$ such that

$$\begin{aligned} \int_0^1 g(h(y)) \cdot \frac{1}{2} (dF^{0,l}(y) + dF^{0,h}(y)) &> \int_0^1 g(h(y)) dy \\ &= \int_0^1 h(y) dy \geq \int_0^1 h(y) \cdot \frac{1}{2} (dF^{0,1}(y) + dF^{0,h}(y)). \end{aligned} \tag{A.10}$$

Take now $h(x) = x^{1/2}$, $g(x) = 2(1 - \delta) \cdot x^4 + \delta \cdot x$, and

$$f^{0,l}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 0 \leq y \leq \epsilon \\ 0 & \text{otherwise} \end{cases},$$

and

$$f^{0,h}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 1 - \epsilon \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

One can easily check that (A.10) holds if we take $\varepsilon > 0$ and $\delta > 0$ arbitrarily low. \square

A.6 Proof of Theorem 3

To show the theorem, several cases have to be distinguished. Before we do so, however, we note that under Assumption 4, an equilibrium must exist with and without globalization (see Proposition 2). Hence, for $\underline{\alpha}_g = 1$ and $\bar{\alpha}_h = 0$ the result holds trivially. Moreover, for $\bar{\alpha}_h = 1$ and $\underline{\alpha}_g = 0$, the desired result follows immediately from Theorem 1 and 2, respectively. The latter imply, respectively, that $\bar{\alpha}_g = 1$ and $\underline{\alpha}_h = 0$ in such cases. Henceforth, we can therefore assume $\underline{\alpha}_g \in (0, 1)$ and $\bar{\alpha}_h \in (0, 1)$. We prove both parts of the theorem.

Proof of Part (i): We stress that we are assuming $\underline{\alpha}_g \in (0, 1)$. From Proposition 1, it must then be that

$$\int_0^1 g(h(y)) dy = \int_0^1 g(h(y)) dF^{\underline{\alpha}_g, m}(y).$$

Then, Assumption 1(iii) and the fact that $g(\cdot)$ is convex imply that

$$\int_0^1 h(y) dy \leq \int_0^1 h(y) dF^{\underline{\alpha}_g, m}(y). \quad (\text{A.11})$$

This follows from Hammond, John S. III. (1974, Theorem 3, Part b.2) along the lines of the proof of Lemma 1. If, on the one hand, (A.11) holds as equality, then by Proposition 1 we know that $\underline{\alpha}_g$ is also an equilibrium without globalization. This immediately implies that $\underline{\alpha}_h \leq \underline{\alpha}_g$. On the other hand, assume that Inequality (A.11) holds strictly. Because of continuity of $\int_0^1 h(y) dF^{\alpha, m}(y)$ in α , there are two options. First,

$$\int_0^1 h(y) dy = \int_0^1 h(y) dF^{\hat{\alpha}, m}(y)$$

for some $\hat{\alpha} \in (0, \underline{\alpha}_g)$. Using Proposition 1, we obtain that $\hat{\alpha}$ is an equilibrium without globalization, and hence $\underline{\alpha}_h \leq \hat{\alpha} < \underline{\alpha}_g$. Second,

$$\int_0^1 h(y) dy < \int_0^1 h(y) dF^{\alpha, m}(y)$$

for all $\alpha \in (0, \underline{\alpha}_g)$. This is equivalent to saying that

$$\int_0^1 h(y)dy > \int_0^1 h(y) \cdot \left(\frac{1}{2}dF^{\alpha,l}(y) + \frac{1}{2}dF^{\alpha,h}(y) \right) \quad (\text{A.12})$$

for all $\alpha \in (0, \underline{\alpha}_g)$. To derive (A.12) we have used Equation (1). Finally, by continuity of expected payoffs in the matching, Condition (15) must hold and NAM must be an equilibrium without globalization. Hence, $\underline{\alpha}_h = 0 < \underline{\alpha}_g$.

Proof of Part (ii): We stress that we are assuming $\bar{\alpha}_h \in (0, 1)$. From Proposition 1, it must then be that

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{\bar{\alpha}_h,m}(y).$$

Given the above equality, Lemma 1 implies that

$$\int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF^{\bar{\alpha}_h,m}(y). \quad (\text{A.13})$$

As before, Inequality (A.13) allows us to distinguish two scenarios. First, assume that

$$\int_0^1 g(h(y))dy = \int_0^1 g(h(y))dF^{\hat{\alpha},m}(y).$$

for some $\hat{\alpha} \in [\bar{\alpha}_h, 1)$. Then, $\hat{\alpha}$ is an equilibrium with globalization, and hence $\bar{\alpha}_h \leq \hat{\alpha} \leq \bar{\alpha}_g$.

Second, assume that

$$\int_0^1 g(h(y))dy > \int_0^1 g(h(y))dF^{\hat{\alpha},m}(y).$$

for all $\alpha \in [\bar{\alpha}_h, 1)$. Continuity of $\int_0^1 g(h(y))dF^{\alpha,m}(y)$ in α then leads to Condition (9). Proposition 1 then implies that with globalization, PAM is an equilibrium. Hence, $\bar{\alpha}_h < 1 = \bar{\alpha}_g$.

□

Online Appendix

B Mathematical Appendix

In this appendix, we provide further details for some of the discussions in the main text.

B.1 Uniqueness of equilibrium

In this appendix, we discuss uniqueness of the equilibrium. In particular, we show that, given Assumption 4, NAM is the unique equilibrium if and only if

$$V(t^m|\mu(\alpha)) > V(t^m|\mu(0)) \text{ for all } \alpha \in (0, 1], \quad (\text{B.1})$$

while PAM is the unique equilibrium if and only if

$$V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha)) > V(t^l|\mu(1)) + V(t^h|\mu(1)) \text{ for all } \alpha \in [0, 1). \quad (\text{B.2})$$

Condition (B.1) requires that mixed teams expect the lowest payoff when all teams are mixed, while Condition (B.2) requires that the average payoff of a high- and a low-skilled team is lowest when no team is mixed.⁴⁰

To see that NAM (PAM) is the unique equilibrium if and only if Condition (B.1) (Condition (B.2)) is satisfied, note first that in our symmetric setup in which only skills are relevant for ranking, in the case where all teams are mixed, they should all expect to be ranked in any position with equal probability. In fact, from Equation (1), it must be that

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{0,m}(y) = V(t^m|\mu(0)).$$

On the one hand, assume Condition (B.1). Then, for all $\alpha \in (0, 1]$,

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) < V(t^m|\mu(\alpha)) = \int_0^1 h(y)dF^{\alpha,m}(y).$$

This means that there cannot be an equilibrium with $\alpha \in (0, 1]$ —see Proposition 1—, and hence an equilibrium with NAM ($\alpha = 0$) is the only possibility. Conversely, suppose Condition (B.1) does not hold. Then, by Assumption 4, there either must be some $\alpha \in (0, 1]$ such that

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) = V(t^m|\mu(\alpha)) = \int_0^1 h(y)dF^{\alpha,m}(y)$$

⁴⁰There can also be a unique equilibrium with $\alpha \in (0, 1)$. Our main focus is on equilibria with PAM and NAM, respectively, and we thus limit our attention to these cases.

and this α corresponds to an equilibrium—see Proposition 1—or it must be that

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) > V(t^m|\mu(1)) = \int_0^1 h(y)dF^{1,m}(y),$$

in which case PAM is an equilibrium. In either case, NAM cannot be the only equilibrium.

Next, assume Condition (B.2). Then, it must hold that, for all $\alpha \in [0, 1]$,

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] < \frac{1}{2} \cdot [V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)]. \end{aligned}$$

Accordingly, there cannot be an equilibrium with $\alpha \in [0, 1)$, and an equilibrium with PAM ($\alpha = 1$) is the only possibility. First, the fact that there cannot be an equilibrium with NAM follows immediately from Proposition 1. Second, the fact that there cannot be an equilibrium with $\alpha \in (0, 1)$ follows from the fact that for $\alpha \in (0, 1)$

$$\int_0^1 h(y)dy < \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)]$$

implies that

$$\int_0^1 h(y)dy > \int_0^1 h(y)dF^{\alpha,m}(y)$$

by Equation (1), and from Proposition 1. Finally, suppose that Condition (B.2) does not hold. Then, by Assumption 4, it either must be that for some $\alpha \in [0, 1)$

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] = \frac{1}{2} \cdot [V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)], \end{aligned}$$

and the same reasoning as the one just shown implies that this α must be an equilibrium, or it must be that

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] > \frac{1}{2} \cdot [V(t^l|\mu(0)) + V(t^h|\mu(0))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{0,l}(y) + dF^{0,h}(y)], \end{aligned}$$

in which case NAM must be an equilibrium. In either case PAM cannot be the only equilibrium.

B.2 Details on the discussions of Section 6.2

In this appendix, we show how Lemma 1 can be applied to the variant of our model of Section 6.2 to show a result analogous to Theorem 1. We have $\tilde{B}^\alpha(\varphi)$ denote the aggregate distribution of productivities in an economy with matching α

$$\tilde{B}^\alpha(\varphi) := \frac{\alpha}{2}B^l(\varphi) + \frac{\alpha}{2}B^h(\varphi) + (1 - \alpha)B^m(\varphi),$$

and $\tilde{b}^\alpha(\varphi) := \frac{d\tilde{B}^\alpha(\varphi)}{d\varphi}$ the associated PDF. With this notation, we have that positive assortative matching is an equilibrium if and only if

$$\int_{\varphi \in \Phi} \pi^1(\varphi) dB^m(\varphi) \leq \int_{\varphi \in \Phi} \pi^1(\varphi) d\tilde{B}^1(\varphi),$$

which is again a supermodularity condition. Using $y := \tilde{B}^1(\varphi)$, we obtain from integration by substitution

$$\int_0^1 \pi^1(\tilde{B}^{1,-1}(y)) \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))} dy \leq \int_0^1 \pi^1(\tilde{B}^{1,-1}(y)) dy,$$

where, as before, we use a superscript -1 to denote an inverse function and where we have simplified the exposition by assuming that $\tilde{b}^1(\varphi)$ has continuous support. $f^{1,m}(y) := \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))}$ is a PDF on $[0, 1]$ that satisfies

$$\mathcal{A} := \{y \in [0, 1] : f^{1,m}(y) \geq 1\}$$

is a convex and compact set by Assumption 1''.⁴¹ Moreover, subtracting the constant $\pi^1(\tilde{B}^{1,-1}(0))$ from both sides, we obtain

$$\int_0^1 \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))} dy \leq \int_0^1 \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] dy,$$

where $h(y) := \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] \in \mathcal{H}$. Hence, we can indeed apply Lemma 1 to generalize Theorem 1.

B.3 Details on the discussions of Section 7.1.2

In this appendix, we show that in case of a patent race as described in Section 7.1.2, the rank distribution of a mixed team satisfies Assumption 1 for any $\lambda^l, \lambda^m, \lambda^h > 0$.

⁴¹Note that

$$\int_0^1 f^{1,m}(y) dy = \int_{\varphi \in \Phi} \frac{b^m(\varphi)}{\tilde{b}^1(\varphi)} \cdot \tilde{b}^1(\varphi) d\varphi = 1.$$

Note first that the rank of a team with skill level k is distributed according to the cumulative distribution $F^{\alpha,k}(y)$, where for all $y \in [0, 1]$,

$$F^{\alpha,k}(y) = \Pr[r^\alpha(z) \leq y] = \Pr[z \geq r^{\alpha,-1}(y)] = 1 - B^k(r^{\alpha,-1}(y)).$$

The second equality follows from $r^{\alpha'}(\cdot) < 0$. Differentiating with respect to y and applying the chain rule, we obtain

$$f^{\alpha,k}(y) = -B^k(r^{\alpha,-1}(y)) \cdot \frac{d}{dy}r^{\alpha,-1}(y) \quad (\text{B.3})$$

$$= -\lambda^k \exp(-\lambda^k r^{\alpha,-1}(y)) \cdot \frac{d}{dy}r^{\alpha,-1}(y). \quad (\text{B.4})$$

Now, we want to show that the set

$$\mathcal{A} := \{y \in [0, 1] : f^{\alpha,m}(y) \geq 1\}$$

is compact and convex. To make progress, note first that this set is equal to

$$\mathcal{A} = \left\{y \in [0, 1] : \tilde{f}(y) \leq 1\right\},$$

where

$$\begin{aligned} \tilde{f}(y) &:= \frac{(\alpha/2)f^{\alpha,l}(y) + (\alpha/2)f^{\alpha,h}(y) + (1-\alpha)f^{\alpha,m}(y)}{f^{\alpha,m}(y)} \\ &= \frac{(\alpha/2)\lambda^l \exp(-\lambda^l r^{\alpha,-1}(y)) + (\alpha/2)\lambda^h \exp(-\lambda^h r^{\alpha,-1}(y)) + (1-\alpha)\lambda^m \exp(-\lambda^m r^{\alpha,-1}(y))}{\lambda^m \exp(-\lambda^m r^{\alpha,-1}(y))}. \end{aligned}$$

The equivalence follows from the fact that the numerator in the first row is equal to one by Equation (1). The advantage of expressing the set \mathcal{A} this way is that it allows us to get rid of the term $\frac{d}{dy}r^{\alpha,-1}(y)$ in Equation (B.3). Now, because $r^{\alpha,-1}(y)$ is strictly decreasing, showing that the bounded set $\left\{y \in [0, 1] : \tilde{f}(y) \leq 1\right\}$ is closed and convex is equivalent to proving that the set

$$\left\{z \in [0, \infty) : \tilde{f}(z) \leq 1\right\}$$

is closed and convex, where $z = r^{\alpha,-1}(y)$. The desired result then follows from noting that

$$\frac{d^2}{(dz)^2}\tilde{f}(z) = \frac{\alpha}{2} \left[\frac{\lambda^l}{\lambda^m} (\lambda^m - \lambda^l)^2 \exp[(\lambda^m - \lambda^l)z] + \frac{\lambda^h}{\lambda^m} (\lambda^m - \lambda^h)^2 \exp[(\lambda^m - \lambda^h)z] \right] > 0.$$

This implies that $\tilde{f}(z)$ can cross one at most once from below.

B.4 Details on the discussions of Section 7.2.2

In this appendix, we show that in the variant of the Melitz (2003)-model considered in Section 7.2.2, conditional on the matching, the mass of entrants is independent of the trade environment.

The productivity of an entrepreneurial team t^k with skill level $k \in \{l, m, h\}$ is drawn from a Pareto distribution with skill-dependent minimum-productivity level

$$b^k(\varphi) = \begin{cases} \frac{\gamma \varphi^{k\gamma}}{\varphi^{\gamma+1}} & \text{if } \varphi \geq \underline{\varphi}^k \\ 0 & \text{otherwise} \end{cases}, \quad (\text{B.5})$$

and where $\underline{\varphi}^l \leq \underline{\varphi}^m \leq \underline{\varphi}^h$. Ignoring knife-edge cases, free entry implies that the (expected) income of the lowest skilled entrepreneurs must equal the wage rate of workers, while higher skilled entrepreneurs earn positive rents. Note that this immediately implies that all high-skilled agents must work as entrepreneurs if some low-skilled agents are to work as entrepreneurs as well.

Now, to simplify the exposition, suppose that entrepreneurial talent is scarce such that there are always some low-skilled teams.⁴² In equilibrium, free entry then implies that the ex-ante expected profit of a low-skilled team has to be equal to twice the wage rate for workers

$$\sum_{j \in \mathcal{I}} \int_{\varphi_{ij}^*}^{\infty} [\pi_{ij}(\varphi) - f_{ij}w_i] b^l(\varphi) d\varphi = 2w_i, \quad (\text{B.6})$$

where \mathcal{I} is the set of countries, f_{ij} are the fixed cost in terms of domestic labor of serving destination country j from country i , w_i is the wage rate in country i , and $\pi_{ij}(\varphi)$ are the variable profits that a firm in country i with productivity φ can make when serving consumers in destination country j . φ_{ij}^* is the well-known productivity cutoff, i.e. a firm in country i with productivity φ_{ij}^* just breaks even when serving destination country j . In equilibrium, a firm in country i will serve destination j if and only if it has productivity $\varphi \geq \varphi_{ij}^*$. The labor market in country i clears if

$$M_{ei} \left[\sum_{j \in \mathcal{I}} \int_{\varphi_{ij}^*}^{\infty} [l_{ij}^v(\varphi) + f_{ij}] b_i(\varphi) d\varphi \right] + M_{ei} f_{ei} = L_i, \quad (\text{B.7})$$

where M_{ei} is the total mass of entering firms, $l_{ij}^v(\varphi)$ the variable labor input for a firm in country i with productivity φ associated with serving destination country j , and f_{ei} is the

⁴²That is, we consider the case where $M_{ei} > \frac{L_i^h}{2}$, where L_i^h denotes the mass of high-skilled labor in country i . This restriction is not essential for the following arguments and it can easily be dispensed with at the expense of additional notational complexity.

fixed cost of labor involved with founding a firm, i.e. $f_{ei} = 2$ if entrepreneurial teams have two team members. $b_i(\varphi)$ is the productivity distribution of all firms in country i , which depends on the matching of entrepreneurs to teams. In particular, let α_i^k , $k \in \{l, m, h\}$, be the share in country i of entrepreneurial teams with skill level k . With this notation, we have that

$$b_i(\varphi) = \alpha_i^l b^l(\varphi) + \alpha_i^m b^m(\varphi) + \alpha_i^h b^h(\varphi). \quad (\text{B.8})$$

Combining Equations (B.5) to (B.8) and following steps as shown in the online appendix of Melitz and Redding (2014), we get⁴³

$$M_{ei} = \frac{L_i}{f_{ei}} \left[1 + \frac{\sigma(\gamma - 1) + 1}{\sigma - 1} \cdot \frac{\alpha_i^l \underline{\varphi}^{l\gamma} + \alpha_i^m \underline{\varphi}^{m\gamma} + \alpha_i^h \underline{\varphi}^{h\gamma}}{\underline{\varphi}^{l\gamma}} \right]^{-1}. \quad (\text{B.9})$$

Hence, indeed, the mass of entering firms does not directly depend on the trade environment.

⁴³For the case of $\underline{\varphi}^l = \underline{\varphi}^m = \underline{\varphi}^h$, Expression (B.9) reduces to Equation (22) in Melitz and Redding (2014).