

the rank distribution of the mixed teams is biased towards achieving mid-range ranks, while low-skilled teams (high-skilled teams) are biased towards achieving low-range ranks (high-range ranks). In other words, teams whose members are matched positively assorted will (on average) benefit more from a globalization-induced amplified ‘superstar effect’.

While Theorem 1 analyzes the existence of equilibria with positive assortative matching, the concentration of talent also increases when moving out of an equilibrium with negative assortative matching. It turns out that globalization also promotes the concentration of talent in this latter sense, as shown next.

Theorem 2

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then,

- (i) if $(\mathcal{W}, g \circ h, \mathcal{F})$ satisfies NAM, so does $(\mathcal{W}, h, \mathcal{F})$;
- (ii) sometimes $(\mathcal{W}, h, \mathcal{F})$ satisfies NAM and $(\mathcal{W}, g \circ h, \mathcal{F})$ does not.

Proof:

See Appendix A.5. □

Clearly, PAM ($\alpha = 1$) and NAM ($\alpha = 0$) represent two extremes of the one-dimensional space $\{\mu(\alpha)\}_{\alpha \in [0,1]}$. Theorems 1 and 2 are therefore *local*, in the sense that they are informative about one value of α only. More generally, one could ask whether, as a result of globalization, the concentration of talent increases by shifting the economy from an equilibrium with $\mu(\alpha)$ to an equilibrium with $\mu(\alpha')$, where $\alpha, \alpha' \in [0, 1]$ satisfy $\alpha \leq \alpha'$. In general, this depends on the (global) behavior of $V(t^m | \mu(\alpha))$ under payoff schemes $h(\cdot)$ and $g(h(\cdot))$. Yet, our setup allows for predictions regarding the equilibria with lowest and highest concentration of talent, respectively, if we impose two conditions: First, a team’s expected payoff depends continuously on the economy-wide matching, as covered by Assumption 4. This is a rather mild assumption and mainly of technical nature—we refer to Section 4.2. Second, a mixed team is more likely to achieve mid-range ranks also in economies where not all other teams are matched positively assorted. This is formalized next.

Assumption 1 (continued)

(iii) The set

$$\mathcal{A}_\alpha^{\mathcal{F}} := \{y \in [0, 1] : f^{\alpha,m}(y) \geq 1\}$$

is convex and compact for all $\alpha \in (0, 1)$.

Assumption 1(iii) is a natural extension of parts (i) and (ii), and its rationale has already been discussed in Section 3.

Now, let $\underline{\alpha}_h$ ($\bar{\alpha}_h$) denote the equilibrium with lowest (highest) concentration of talent prior to globalization and, analogously for $\underline{\alpha}_g$ ($\bar{\alpha}_g$). The latter refers to the case after globalization. With this notation, we obtain the following result regarding the global effect of globalization on the concentration of talent:

Theorem 3

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then, under Assumption 4,

(i) $\underline{\alpha}_h \leq \underline{\alpha}_g$

(ii) $\bar{\alpha}_h \leq \bar{\alpha}_g$.

Proof:

See Appendix A.6.

□

Theorem 3 shows that our setup allows for predictions regarding the effect of globalization on the concentration of talent above and beyond equilibria with positive and negative assortative matching. Specifically, it says that no matter the set of equilibria that exist with and without globalization, it must always be that the equilibrium with highest (lowest) concentration of talent is one with (without) globalization, and this must in particular be true if we focus on *mixed equilibria* beyond PAM and NAM.²²

It is worth noting that Theorem 3 further allows addressing a potential caveat pertaining to Theorem 1. In the latter result, we limit attention to the existence of equilibria with positive assortative matching, without considering uniqueness. Hence, one might wonder whether there might be a unique equilibrium with positive assortative matching prior to globalization, but such an equilibrium might not be unique with globalization. It turns out that under the mild regularity conditions underlying Theorem 3 this cannot occur, as it would contradict Theorem 3(i).

To sum up, Theorems 1 to 3 reveal that our theory predicts strong implications of globalization for the concentration of talent. This has important distributional consequences, as we discuss next.

²²In Theorem 3, $\underline{\alpha}_h, \underline{\alpha}_g, \bar{\alpha}_h, \bar{\alpha}_g \in [0, 1]$. Hence, under Assumption 1 and Assumption 4, Theorem 3 implies the results of Theorems 1 and 2. We stress, however, that Assumption 4 is not needed for Theorems 1 and 2.

5.2 Globalization and wage inequality

Although our main interest is to understand how globalization impacts matching outcomes, it is worth mentioning that a globalization-induced increased concentration of talent has distributional consequences. It is well known that conditional on positive assortative matching, globalization can increase (decrease) *relative* wages of high-skilled (low-skilled) workers via amplified superstar effects.²³ While under plausible restrictions this is also the case here, our setup is general enough to accommodate situations where this direct effect of globalization on wages might not take place. More importantly, there is an additional, indirect effect, since the matching might also change as a result of globalization. To see this, note that we can decompose the overall wage effect for low-skilled workers as follows

$$\begin{aligned}
 & \frac{1}{2} \cdot \int_0^1 g(h(y)) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_h, l}(y) \\
 &= \left[\frac{1}{2} \cdot \int_0^1 g(h(y)) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_g, l}(y) \right] \\
 &+ \left[\frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_g, l}(y) - \frac{1}{2} \cdot \int_0^1 h(y) dF^{\alpha_h, l}(y) \right],
 \end{aligned} \tag{20}$$

and analogously for the wage of the high-skilled workers. In the above equation, we have used α_g (α_h) to denote the matching outcome with (without) globalization, and we have assumed that $\alpha_g \geq \alpha_h > 0$, for simplicity, so that there is no NAM. The first term on the right-hand side of Equation (20) captures the effect of globalization conditional on the matching. For $\alpha_g = 1$ it corresponds to the effect previously considered in the literature. The second term in brackets captures the additional effect that arises from a potential change in the matching, i.e. it is the change in the wage of the low-skilled workers conditional on the payoff scheme.

Now, recall that for $\alpha > 0$ the equilibrium conditions of Proposition 1 refer to the payoff distribution of mixed teams, but do not directly refer to the payoff distributions of high-skilled teams or low-skilled teams. The latter distributions nonetheless determine the wages. This means that the sign of both terms on the right-hand side of Equation (20) is ambiguous, unless more structure is imposed. Again, this is due to the generality of our setup. Yet, plausible restrictions allow for stronger comparative statics. An interesting case in this regard is one where $F^{\alpha_h, l}(y) \leq F^{\alpha_g, l}(y)$ for all $y \in [0, 1]$, i.e. $F^{\alpha_h, l}(y)$ first-order stochastically dominates distribution $F^{\alpha_g, l}(y)$ (Mas-Colell et al., 1995, Prop. 6.D.1). In this case, the second term on the right-hand side of Equation (20) is negative. This suggests that in a

²³Cf. the literature review in Section 2.

world where low-skilled teams are relatively better at competing against mixed teams than against assortatively matched teams, it is more natural to expect that the net effect of globalization on relative wages of the low-skilled workers is negative above and beyond any potential effect conditional on the matching.²⁴

6 Extensions and Alternative Specifications

In this section, we discuss several extensions of our baseline setup and show that our main result—Theorem 1—readily extends to these alternative settings.

6.1 Several types and team members

We start by analyzing how Theorem 1 can be extended to the case with several skill types and several team members. Suppose there are $S \geq 2$ types with arbitrary population shares, which can be identified by their skill level $s \in \mathcal{S}$, where \mathcal{S} denotes the set of skills available in the economy. Let $N \geq 2$ denote the number of workers in each team t , with $\mathcal{S}_t = \{s_t^1, s_t^2, \dots, s_t^N\}$ denoting the multiset of skill levels in team t . Then, there is an equilibrium with positive assortative matching if and only if for every worker type $s \in \mathcal{S}$,

$$V(t^{\{s\}^N} | PAM) - (N - 1) \cdot w^s \geq \max_{\hat{\mathcal{S}} \in \mathcal{S}^{N-1}} \left\{ V(t^{s \cup \hat{\mathcal{S}}} | PAM) - \sum_{\hat{s} \in \hat{\mathcal{S}}} w^{\hat{s}} \right\}.$$

Here, w^s denotes the equilibrium wage for a worker with skill level s , $t^{\mathcal{S}_t}$ identifies a team with workers of skill levels \mathcal{S}_t , and

$$V(t^{\mathcal{S}_t} | PAM) = \int_0^1 h(y) dF^{PAM, \mathcal{S}_t}(y)$$

indicates the expected payoff of team $t^{\mathcal{S}_t}$, assuming that all other workers are matched positively assorted. As in the previous sections, $F^{PAM, \mathcal{S}_t}(y)$ is the rank distribution of a team t with skill levels \mathcal{S}_t when competing against teams that are arranged according to PAM, and $f^{PAM, \mathcal{S}_t}(y) := \frac{dF^{PAM, \mathcal{S}_t}}{dy}(y)$ is the associated PDF.²⁵ It can be verified that wages in an equilibrium with positive assortative matching satisfy

$$w^s = \frac{1}{N} \cdot V(t^{\{s\}^N} | PAM), \text{ for all } s \in \mathcal{S}.$$

²⁴An increased concentration of talent may well have distributional consequences above and beyond any immediate wage effect, e.g. in the presence of knowledge spillovers. A thorough investigation of such effects is beyond the scope of our paper and is left for future research.

²⁵We assume that for all $\mathcal{S}_t \in \mathcal{S}^N$, f^{PAM, \mathcal{S}_t} satisfies all the regularity conditions imposed on $f^{\alpha, k}$ in Section 3.

Using the above equilibrium wages and rearranging terms, we obtain that there is an equilibrium with positive assortative matching if and only if

$$\frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} V(t^{\{s\}^N} | PAM) \geq V(t^{\mathcal{S}_t} | PAM), \text{ for all } \mathcal{S}_t \in \mathcal{S}^N. \quad (21)$$

This is again a supermodularity condition.

Now, consider the following adaptation of Assumption 1(i):

Assumption 1'

For any multiset of skills $\mathcal{S}_t \in \mathcal{S}^N$, it holds that

$$\mathcal{A} := \left\{ y \in [0, 1] : f^{PAM, \mathcal{S}_t}(y) \geq \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) \right\} \quad (22)$$

is a convex and compact set.

Analogously to the case of two types and two team members, the most natural interpretation of Condition (22) is that the rank distributions of mixed teams are biased towards achieving mid-range ranks when compared to assortatively matched teams of the types corresponding to its team members. For a given $\mathcal{S}_t \in \mathcal{S}^N$, we can rewrite Condition (21) as

$$\int_0^1 h(y) \cdot \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) dy \geq \int_0^1 h(y) \cdot f^{PAM, \mathcal{S}_t}(y) dy.$$

Letting $F(y) := \int_0^y \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(x) dx$ and using the change of variables $z := F(y)$ yields

$$\int_0^1 h(F^{-1}(z)) dz \geq \int_0^1 h(F^{-1}(z)) \cdot \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} dz,$$

where $f(y) := \frac{d(F(y))}{dy}$.²⁶ Clearly, $\frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))}$ is a PDF on $[0, 1]$,²⁷ and

$$\mathcal{A} := \left\{ z \in [0, 1] : \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} \geq 1 \right\}$$

is a convex and compact set by Assumption 1'. Hence, we can apply Lemma 1 to generalize Theorem 1 to the case with many types and team members.

²⁶We have simplified the exposition by assuming that $f(\cdot)$ has continuous support. At the expense of additional notational complexity, this simplification can easily be dispensed with.

²⁷In particular, $\frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))}$ is positive and satisfies

$$\int_0^1 \frac{f^{PAM, \mathcal{S}_t}(F^{-1}(z))}{f(F^{-1}(z))} dz = \int_0^1 f^{PAM, \mathcal{S}_t}(y) dy = 1.$$

6.2 Beyond rank competition

Thus far we have considered economies with competition for rank. In such cases, there is a clear distinction between rank-dependent payoffs that are affected by globalization, on the one hand, and the competition and matching outcomes that impact teams' rank distributions, on the other. The focus on rank competitions therefore allows us to discuss our main mechanisms of interest in a transparent way. Yet, our main results also apply to models where a team's payoff directly depends on its own productivity and the productivities of all other teams in the economy, as we show next. To simplify the exposition, we return to considering two types of workers only—high-skilled and low-skilled—that form teams of two, but arguments along the lines of Section 6.1 imply that the results shown extend to scenarios with more types and team members.

Let us assume that each team forms and then receives a random productivity draw φ from a publicly-known, skill-dependent probability distribution $B^k(\varphi)$, $k \in \{l, m, h\}$, that has support $\Phi \subseteq \mathbb{R}_+$, with density function $b^k(\varphi) := \frac{dB^k(\varphi)}{d\varphi}$. After receiving their productivity draw, teams compete in a market where each team's payoff depends on its own productivity and the productivities of all other teams in the economy, analogously to e.g. a Melitz (2003)-model (see Section 7.2.2 for a discussion). These productivities depend on the (whole) matching. With a continuum of workers, however, there is no aggregate uncertainty about the distribution of productivities in the economy, once the matching is given. Hence, we can summarize the entire distribution of productivities in the economy by a parameter α . As in Section 3, it denotes the share of teams whose members are matched positively assorted. Accordingly, we use $\pi^\alpha(\varphi)$ to denote the payoff of a team with productivity φ given matching α . Then, the expected payoff of a team $k \in \{l, m, h\}$ given PAM is

$$V(t^k | \mu(1)) = \int_{\varphi \in \Phi} \pi^1(\varphi) dB^k(\varphi),$$

and PAM is an equilibrium if and only if

$$\int_{\varphi \in \Phi} \pi^1(\varphi) dB^m(\varphi) \leq \int_{\varphi \in \Phi} \pi^1(\varphi) \cdot \frac{1}{2} \cdot [dB^l(\varphi) + dB^h(\varphi)].$$

Now, consider the following adaptation of Assumption 1:

Assumption 1''

The set

$$\mathcal{A} := \left\{ \varphi \in \Phi : b^m(\varphi) \geq \frac{1}{2} [b^l(\varphi) + b^h(\varphi)] \right\} \quad (23)$$

is convex and compact.

Then—as we show in Appendix B.2—we can again use Lemma 1 to show that Theorem 1 generalizes to this case for any form of globalization, $g(\cdot)$, that is an increasing, convex transformation of payoffs conditional on PAM, $\pi^1(\varphi)$.

6.3 Migration

We conclude this section with a brief discussion of migration. In our analysis, we have taken the pool of workers as given and, in particular, we have assumed that it is itself not affected by globalization. While in many instances this may be a reasonable approximation given that capital, goods, and services tend to be more mobile than people, in other cases—such as our illustrative example from European football—workers tend to be more mobile. It is thus relevant to discuss how our results are affected by labor mobility. To that end, we consider two polar cases.

First, assume that there are two perfectly symmetric economies that were initially separated and then integrated their labor markets, but which still have separate competitions. In the case of European football, for example, teams still compete in their national leagues. Our results remain valid in this first polar approach to labor mobility. That is, globalization in the form of a convex transformation of payoffs increases the concentration of talent with this extreme form of labor mobility as well. This is because with perfectly mobile labor, wages for high- and low-skilled workers have to be the same across countries. Hence, migration will not change the skill composition of the two economies. Referring to our football example, indeed it is not the case that the very best players all play in one country.

Our second polar case regarding labor mobility assumes that the two countries will integrate completely, i.e. both their competition and their labor markets will be merged. This corresponds to a simple scaling of our economy, and our results directly apply as long as the competition and the payoff structure are scale invariant.²⁸

7 Micro-foundations

In the previous sections, we have presented a simple reduced-form analysis of how globalization impacts the concentration of talent across competing teams. In this section, we present

²⁸In this case, integrating the economies yields a payoff scheme $\tilde{h}(y)$, with $\tilde{h}(2y) := h(y)$, and a rank distribution for a team $k \in \{l, m, h\}$ in an economy where a share α of teams have members who are matched positively assorted $\tilde{f}^{\alpha,k}(y)$, where $\tilde{f}^{\alpha,k}(2y) := f^{\alpha,k}(y)$. A simple change of variables $x := 2y$ then implies that integration will not impact equilibrium outcomes.

micro-foundations for our two main assumptions: Assumption 1 (about competition) and Assumption 3 (about globalization). We begin with the former, which specifically refers to how the competition mode determines the ranking, and then consider the latter, which specifically refers to how globalization impacts payoffs.

7.1 Micro-foundations for the competition mode

7.1.1 Head-to-head competition

Suppose that all teams compete head to head and are then ranked according to the number of victories. This is in line with competition in a sports league, for example. More specifically, assume that each time two teams meet, there is a fixed probability $p > \frac{1}{2}$ that the higher skilled team wins, with that probability being $\frac{1}{2}$ if two equally skilled teams compete. Then, as the number of games that a particular team is involved in goes to infinity, the ratio of victories for this team will converge to its expected value by the law of large numbers.²⁹ As a consequence, the rank distribution of a low-skilled team will be uniform on $[0, \frac{\alpha}{2}]$, that of a mixed team will be uniform on $[\frac{\alpha}{2}, 1 - \frac{\alpha}{2}]$ and that of a high-skilled team uniform on $[1 - \frac{\alpha}{2}, 1]$.³⁰ This implies that the set $\mathcal{A}_\alpha^{\mathcal{F}}$ is (asymptotically) convex and compact, in which case Assumption 1 is satisfied.

7.1.2 Patent race

Assumption 1 also arises naturally when teams compete against each other in a patent or innovation race. To show this, assume that after teams have formed, they are ranked according to the timing of first events drawn from a Poisson process with skill-dependent arrival rate $\lambda^k > 0$, with $k \in \{l, m, h\}$. That is, the cumulative distribution function for the time of invention of a team with skill level λ^k is³¹

$$B^k(z) = 1 - e^{-\lambda^k z}.$$

²⁹We follow the convention in the economics literature and apply the law of large numbers to a continuum of random variables.

³⁰To see this note that the expected share of victories for a low-skilled team will be $p^l = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot (1 - p)$, that of a mixed team will be $p^m = \frac{\alpha}{2} \cdot p + \frac{\alpha}{2} \cdot (1 - p) + \frac{1 - \alpha}{2} = \frac{1}{2}$, and that of a high-skilled team will be $p^h = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot p$. It is then straightforward to verify that for every $p > \frac{1}{2}$, we must have $p^l < p^m < p^h$. The resulting rank CDFs are not continuously differentiable. Note, however, that we could easily smoothen things by assuming that the overall skill level of a team is itself subject to a random shock such that some teams of low-skilled workers, for example, end up being relatively high skilled.

³¹See Loury (1979) and Dasgupta and Stiglitz (1980) for seminal contributions using Poisson processes in the modeling of patent races.

As usual, we let $b^k(z)$ denote the corresponding PDF. In an economy with a share α of all teams whose members are matched positively assorted, the (expected) rank $y \in [0, 1]$ of a team that invents at some time $z \in \mathbb{R}_+$ is then given by

$$y = r^\alpha(z) = \frac{\alpha}{2}e^{-\lambda^l z} + \frac{\alpha}{2}e^{-\lambda^h z} + (1 - \alpha)e^{-\lambda^m z},$$

where $(\alpha/2)e^{-\lambda^l z}$ is the measure of low-skilled teams that have not yet innovated at time z and analogously for $(\alpha/2)e^{-\lambda^h z}$ and $(1 - \alpha)e^{-\lambda^m z}$. As we show in Appendix B.3, the rank distribution of the mixed team satisfies Assumption 1 for any $\lambda^l, \lambda^m, \lambda^h > 0$. Of course, if skills are to be meaningful, it should be $\lambda^l \leq \lambda^m \leq \lambda^h$.

7.1.3 Pareto distribution of team's productivity

Finally, we show that Assumption 1'' is satisfied if a team's productivity is drawn from a Pareto distribution with skill-dependent location parameter. More specifically, given $\gamma > 0$ let

$$b^k(\varphi) := \begin{cases} \frac{\gamma\varphi^{k\gamma}}{\varphi^{\gamma+1}} & \text{if } \varphi \geq \varphi^k \\ 0 & \text{otherwise} \end{cases}$$

be the PDF for a team with skill level $k \in \{l, m, h\}$, and then suppose that $\underline{\varphi}^l \leq \underline{\varphi}^m \leq \underline{\varphi}^h$. It immediately follows that the set

$$\mathcal{A} := \left\{ \varphi \in \Phi : b^m(\varphi) \geq \frac{1}{2} [b^l(\varphi) + b^h(\varphi)] \right\}$$

is convex. Hence, Assumption 1'' is satisfied.³²

7.2 Micro-foundations for globalization as convex transformation

To rationalize Assumption 3, we build on the models of Rosen (1981) and Melitz (2003), respectively.

³²If $\underline{\varphi}^{m\gamma} \geq \frac{1}{2} [\underline{\varphi}^{l\gamma} + \underline{\varphi}^{h\gamma}]$, we have $\mathcal{A} = [\underline{\varphi}^m, \infty)$. If $\underline{\varphi}^{m\gamma} < \frac{1}{2} [\underline{\varphi}^{l\gamma} + \underline{\varphi}^{h\gamma}]$, we have $\mathcal{A} = [\underline{\varphi}^m, \underline{\varphi}^h)$. These sets are not compact. Nevertheless, the former is compact when considering a truncated Pareto distribution with truncation threshold $\bar{\varphi}$, and we can approximate the Pareto distribution by choosing $\bar{\varphi}$ large. The latter is compact when considering the following distribution for the high-skilled team

$$b^h(\varphi) = \frac{1}{\epsilon} \cdot \int_{-\epsilon/2}^{\epsilon/2} b_\delta^h(\varphi) d\delta,$$

where $b_\delta^h(\varphi)$ denotes the Pareto distribution with lower-bound $\underline{\varphi}_\delta^h := \underline{\varphi}^h + \delta$. Again, by choosing $\epsilon \rightarrow 0$ we can approximate the Pareto distribution.

7.2.1 Rosen (1981) with competing teams

In his seminal paper, Rosen (1981) shows how small differences in talent can result in large differences in sales and income at the top in the case of markets with imperfect substitutability of quantity for quality. Moreover, he shows that globalization—namely, an increase in the size of the market—results in a convex transformation of payoffs. In this section, we briefly discuss a simple variant of his model with competing teams and show how it maps into our reduced-form analysis.

Suppose that consumers demand overall services $x := nz$ from competing teams, where z is the quality of service provided by a team and n is the quantity consumed, such as the number of games attended. Consumers face a fixed cost s per unit of service consumed, which we can think of as representing e.g. the time cost associated with consuming the service. Hence, the total cost of consuming n units of a service with quality z is equal to $n(p(z) + s)$, where $p(z)$ is the price per unit of service of quality z . The cost per effective unit is $v := (p(z) + s)/z$. The latter cost is constant across teams in equilibrium. Rosen (1981) allows for the existence of internal and external dis-economies of scale, i.e. the cost of providing m units of service, $C(m)$, is increasing and convex. In addition, the quality of service $z = z(y, m)$ is decreasing in the number of units sold, reflecting congestion effects.³³ In this setup, y is a measure of the underlying value of the service provided by a team, and we can allow for different interpretations of y . In what follows, we think of y simply as the rank of the team—reflecting the fact that supporters enjoy seeing their team winning—but it could also express the skill level of the team drawn from a random distribution as discussed in Section 6.2. Rosen (1981) shows that the payoff of a team ranked y , $h(y)$, satisfies

$$h''(y) = v(z_y + mz_{ym}) \frac{\partial m}{\partial y} + vmz_{yy} ,$$

i.e. the revenue function is convex over ranks if $z_{yy} \geq 0$ and $z_{ym} \geq 0$, where the latter inequality implies that higher ranked teams are better at serving larger audiences. The intuition is that in this case higher-ranked teams cannot only charge a higher price, but it is also profitable for them to serve larger audiences, which implies a convex payoff-scheme. The important point to note is that convexity increases in v , which is the market price per unit of service. That is, when in the wave of globalization demand for these services goes up so that v increases, the payoff scheme becomes more convex, i.e. Assumption 3 holds. Hence, our reduced-form analysis applies to this variant of the Rosen (1981) model, provided that the competition satisfies Assumption 1.

³³For example, Rosen (1981) suggests that it is more valuable to attend a concert in a small concert hall than in the Yankee Stadium.

7.2.2 Fixed cost of market entry: Melitz (2003)-model with entrepreneurial teams

Assumption 3 is also naturally satisfied if teams have the opportunity to access foreign markets, albeit at a fixed cost. Suppose that the gains from entering a foreign market are increasing with a team's rank in its domestic economy. In the case of European football, we may think of entering a foreign market as actively trying to increase a team's fan base in these markets to raise revenues via sponsoring, merchandising, or licensing, for example. Such endeavors are naturally more promising for teams that perform well in their domestic leagues.³⁴ Now, assume for simplicity that payoffs generated abroad, $\tilde{h}(y)$, are proportional to the domestic payoffs $h(y)$, i.e.

$$\tilde{h}(y) = \lambda \cdot h(y)$$

for some constant $\lambda > 0$. Teams will enter the foreign market only if this is profitable, implying that the total payoff of a team ranked y is

$$g(h(y)) = h(y) + \max\{0, \lambda \cdot h(y) - c\} .$$

It is straightforward to verify that $g(\cdot)$ is increasing and convex.^{35,36}

The same logic also implies that our analysis directly applies to a simple variant of a Melitz (2003)-model with entrepreneurial teams. In this variant, agents are either high-skilled or low-skilled. High-skilled agents are better entrepreneurs, but they have no advantage when employed as a worker. There is an initial stage where agents decide whether or not to become an entrepreneur. Entrepreneurs match to form entrepreneurial teams of two and found a firm. As in the workhorse version of a Melitz (2003)-model, each firm is equipped with a distinct variety, and receives a random productivity draw, φ , from a known Pareto distribution. In this variant, however, the minimum value of φ is increasing in the skill

³⁴In Germany, for example, Bayern Munich and Borussia Dortmund, the biggest and most successful football clubs in recent years, are most actively promoting their teams abroad and they are the only clubs running foreign offices (see <https://www.welt.de/sport/article157261763/Das-Millionenspiel-der-Bundesligaklubs-in-Uebersee.html>). They also have by far the most facebook likes outside of Germany (see <http://media.de/2015/09/23/bundesliga-bis-3-liga-das-grosse-facebook-ranking-der-fussballclubs/>, retrieved on 26 October 2017).

³⁵In this example $g(h(\cdot))$ is not differentiable for all $y \in [0, 1]$. Nevertheless, our results do not hinge crucially on this regularity assumption, which we have only imposed for simplicity.

³⁶We consider the case where globalization gives rise to a convex transformation of teams' payoffs directly. Alternatively, globalization may yield a convex transformation of firm sizes and then spill over to the compensation of managerial teams. Gabaix and Landier (2008), for example, present a model where at the top CEO pay (executive board pay in a simple variant with managerial teams) is proportionate to a power function of firm size, i.e. for all power coefficients larger than or equal to one, a globalization-induced convex transformation of firm sizes with $g(0) = 0$ translates into a convex transformation of executive compensation.

level of the entrepreneurial team. As in the canonical Melitz (2003)-model, firms with productivity draws above some endogenous threshold level will start operating, while all other firms will exit immediately. This implies that under autarky, a firm’s profit is a piecewise linear and convex function of $\varphi^{\sigma-1}$, where $\sigma > 1$ is the constant elasticity of substitution between varieties. The most interesting case is one with selection into exporting, in line with empirical facts. Globalization—a move from autarky to an equilibrium with trade—then implies that the minimum-productivity threshold for firms increases, i.e. the lowest productive firms are forced to exit after a trade liberalization. Firms with intermediate levels of productivity only serve their domestic market, and the most productive firms also export. The key point is that globalization gives rise to a piecewise linear and convex transformation of a firm’s profits as a function of $\varphi^{\sigma-1}$ —we refer to Melitz and Redding (2014) for further details. Moreover, $\varphi^{\sigma-1}$ is Pareto distributed as well, and mixed teams are thus relatively more likely to have mid-range values of $\varphi^{\sigma-1}$, as shown in Section 7.1.3. This implies that this variant of the Melitz (2003)-model reduces to the model analyzed in Section 6.2. We refer to Appendix B.4 for further discussions.³⁷

8 Conclusion

We have investigated the effect of a convex transformation of payoffs on the concentration of talent in large matching markets. We have argued that globalization can attest for this type of transformation of the payoff scheme. This implies that in relative terms, high ranks are rewarded higher prizes *after* globalization than *before*. We have chosen a reduced-form approach to modelling competition that rests on minimal assumptions pertaining to the relationship between skills and market outcomes, and we expect the mechanisms we have considered to be relevant for many actual markets with competing teams. Because payoffs in such markets might be—and often are—influenced by exogenous elements (namely, globalization), our research question clearly seems relevant both theoretically and empirically.

Our main insight is that globalization promotes the concentration of talent, i.e., it may result in the emergence of positive assortative matching. This has important distributional consequences, as it feeds back into the income distribution of modern societies. Potentially

³⁷In the Melitz (2003)-model, the total mass of entering firms is endogenous, which impacts the distribution of skills over entrepreneurial teams. In turn, this mass of entering firms depends on the matching. With a Pareto distribution of firm productivities, however, it does not directly depend on the trade environment. Hence, the endogeneity of the mass of entering firms does not impede the applicability of our main results, as these depend only on the redistributive effects of globalization *conditional* on the matching outcome. See Appendix B.4 for further details.

adverse effects of globalization above and beyond a direct effect on income inequality have received growing attention in recent years (Autor et al., 2014; Che et al., 2016; McManus and Schaur, 2016; Pierce and Schott, 2016). An increased concentration of talent might be an important factor in this regard, because in the presence of learning externalities it may harm low-skilled workers and perpetuate—or even increase—skill differences. More generally, a greater concentration of talent contributes to social segregation. Future work may set out to study such effects and their welfare implications in more detail.

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Appendix

A Proofs

In this Appendix, we prove Propositions 1 and 2, Lemma 1, and Theorems 1–3.

A.1 Proof of Proposition 1

In the following, we show the three parts of Proposition 1.

Proof of Part (i): In an equilibrium with positive assortative matching, $(\mu(1), \underline{w}, \bar{w})$, Conditions (6) reduce to Conditions (7) and (8). Because there is a continuum of workers—and hence every worker can always find another worker with whom to match—it follows that

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,h}(y)$$

and

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,l}(y),$$

which yields Equations (10) and (11). With these expressions for wages, both Conditions (7) and (8) then reduce to the same condition, namely

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) \geq V(t^m|\mu(1)). \quad (\text{A.1})$$

Accordingly, there is an equilibrium with positive assortative matching if and only if (A.1) is satisfied. Now, note that for $\alpha = 1$, Equation (1) reduces to

$$1 = \frac{1}{2} \cdot \frac{dF^{1,l}(y)}{dy} + \frac{1}{2} \cdot \frac{dF^{1,h}(y)}{dy}.$$

Using the above equation and Equation (5), we can write

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) = \int_0^1 h(y) \cdot \left(\frac{1}{2} \cdot dF^{1,l}(y) + \frac{1}{2} \cdot dF^{1,h}(y) \right) = \int_0^1 h(y) dy,$$

and

$$V(t^m|\mu(1)) = \int_0^1 h(y) \cdot dF^{1,m}(y).$$

Finally, using the above two equations in (A.1), we obtain Condition (9). This concludes the proof of Part (i).

Proof of Part (ii): First of all, note that the same argument as in Part (i) implies that in an equilibrium $(\mu(\alpha), \underline{w}, \bar{w})$ with $\alpha \in (0, 1)$, wages are given by (13) and (14). Further, with $\alpha \in (0, 1)$, Conditions (6) reduce to

$$\begin{aligned} V(t^h|\mu(\alpha)) - \bar{w} &= V(t^m|\mu(\alpha)) - \underline{w}, \\ V(t^l|\mu(\alpha)) - \underline{w} &= V(t^m|\mu(\alpha)) - \bar{w}. \end{aligned}$$

Using the equilibrium wages given by Equations (13) and (14) as well as Equation (5), the above two equations can both be rearranged to

$$\frac{1}{2} \cdot V(t^h|\mu(\alpha)) + \frac{1}{2} \cdot V(t^l|\mu(\alpha)) = V(t^m|\mu(\alpha)).$$

Finally, Equation (12) is easily obtained if we use Equation (5) again in combination with Equation (1). This finishes the proof of Part (ii).

Proof of Part (iii): With negative assortative matching, Conditions (6) reduce to

$$V(t^m|\mu(0)) - \bar{w} \geq V(t^l|\mu(0)) - \underline{w}, \quad (\text{A.2})$$

$$V(t^m|\mu(0)) - \underline{w} \geq V(t^h|\mu(0)) - \bar{w}. \quad (\text{A.3})$$

On the one hand, if there is indeed an equilibrium $(\mu(0), \underline{w}, \bar{w})$, it must be that

$$\underline{w} = V(t^m|\mu(0)) - \bar{w}. \quad (\text{A.4})$$

The reason is that wages of both members of a mixed team must add up to total expected payoffs of the team. Using Equation (5) on Equation (A.4), we obtain Equation (17), which is necessary for $(\mu(0), \underline{w}, \bar{w})$ to be an equilibrium. On the other hand, adding Conditions (A.2) and (A.3) yields the following further necessary condition

$$\frac{1}{2} \cdot V(t^h|\mu(0)) + \frac{1}{2} \cdot V(t^l|\mu(0)) \leq V(t^m|\mu(0)). \quad (\text{A.5})$$

Condition (15) follows from using Equations (1) and (5) on (A.5). Finally, Condition (16) follows from noting that (i) if $\bar{w} < \frac{1}{2}V(t^h|\mu(0))$, high-skilled workers would benefit from breaking apart from a mixed team and matching another high-skilled worker, (ii) if $\underline{w} < \frac{1}{2}V(t^l|\mu(0))$, low-skilled workers would benefit from breaking apart from a mixed team and then matching another low-skilled worker. Conversely, for any wages \underline{w} and \bar{w} that satisfy these two latter constraints plus Equation (A.4), no worker can profitably deviate from his current match, implying that the negative assortative matching $(\mu(0), \underline{w}, \bar{w})$ is indeed an equilibrium.³⁸ This finishes the proof of Part (iii).

□

³⁸Note that the set defined in (16) is non-empty by Equation (15).

A.2 Proof of Proposition 2

Observe, on the one hand, that for no equilibrium with PAM to exist, we must have

$$\begin{aligned} \frac{1}{2} \cdot V(t^l|\mu(1)) + \frac{1}{2} \cdot V(t^h|\mu(1)) &= \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{1,h}(y) + \frac{1}{2} dF^{1,l}(y) \right) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) dF^{1,m}(y) = V(t^m|\mu(1)), \end{aligned}$$

where the second equality follows from Equation (1). On the other hand, for no equilibrium with NAM to exist, we must have

$$\begin{aligned} V(t^m|\mu(0)) &= \int_0^1 h(y) dF^{0,m}(y) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{0,h}(y) + \frac{1}{2} dF^{0,l}(y) \right) = \frac{1}{2} \cdot V(t^l|\mu(0)) + \frac{1}{2} \cdot V(t^h|\mu(0)). \end{aligned}$$

Now, suppose that the above inequalities are both satisfied. Then, due to Assumption 4, there must exist $\alpha^* \in (0, 1)$ such that

$$V(t^m|\mu(\alpha^*)) = \frac{1}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{1}{2} \cdot V(t^h|\mu(\alpha^*)).$$

Using Equation (1) as well as the above equation, we obtain

$$\begin{aligned} \int_0^1 h(y) dy &= \frac{\alpha^*}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{\alpha^*}{2} \cdot V(t^h|\mu(\alpha^*)) + (1 - \alpha^*) \cdot V(t^m|\mu(\alpha^*)) = V(t^m|\mu(\alpha^*)) \\ &= \int_0^1 h(y) dF^{\alpha^*,m}(y), \end{aligned}$$

and, hence, there is an equilibrium where workers are arranged according to $\mu(\alpha^*)$. This proves our claim that an equilibrium always exists under Assumption 4.

□

A.3 Proof of Lemma 1

It is convenient to denote the CDF corresponding to a uniform random variable on $[0, 1]$ as

$$I(y) = y, \text{ for all } y \in [0, 1],$$

with $i(y) = 1$ (for all $y \in [0, 1]$) denoting the corresponding PDF. Then, let $F(\cdot)$ be a CDF with support on $[0, 1]$, with $f(\cdot)$ denoting the corresponding PDF, such that

$$\mathcal{A} := \{y \in [0, 1] : f(y) \geq 1\} \tag{A.6}$$

is a convex and compact set, which we can therefore write as $\mathcal{A} = [\underline{a}, \bar{a}]$. We recall that

$$F(t) = \int_0^t f(y)dy. \quad (\text{A.7})$$

We distinguish three cases, for all of which we assume $h \in \mathcal{H}$. We also assume that \mathcal{A} and $[0, 1]$ differ on a set of positive measure, since otherwise we must have $F = I$ and the lemma holds trivially.

Case I: $\underline{a} = 0$ (and $\bar{a} < 1$)

In this case, $I(y)$ first order stochastically dominates $F(y)$. Mas-Colell et al. (1995, Proposition 6.D.1) then implies that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)f(y)dy$$

for all $h \in \mathcal{H}$ (and in particular for $g \circ h$). This means that in this case, the inequalities in Lemma 1 are always satisfied (with or without globalization).

Case II: $0 < \underline{a} < \bar{a} < 1$

From (A.6) and (A.7) and the fact that $F(0) = I(0) = 0$ and $F(1) = I(1) = 1$, it follows that there is $b^* \in (\underline{a}, \bar{a})$ such that

$$F(y) \begin{cases} \leq I(y) = y & \text{if } y \leq b^* \\ = I(y) = y & \text{if } y = b^* \\ \geq I(y) = y & \text{if } y \geq b^* \end{cases}.$$

That is, $F(\cdot)$ crosses $I(\cdot)$ once and from below. In other words, $(I - F)$ has a single change from positive to negative. Moreover, recall that we are assuming $g'(y) \geq g'(0) > 0$ for all $y \in [0, 1]$. This implies that the inverse $g^{-1}(\cdot)$ of $g(\cdot)$ is defined for all $y \in [0, 1]$, and we have

$$\frac{d}{dy}g^{-1}(y) = \frac{1}{g'(y)} > 0,$$

and

$$\frac{d^2}{dy^2}g^{-1}(y) = -\frac{1}{(g'(y))^2} \cdot g''(y) \leq 0.$$

That is, h can be obtained from $g \circ h$ through a concave transformation. From Proposition 6.C.2. in Mas-Colell et al. (1995), we obtain that

$$r_h(y) := -\frac{h''(y)}{h'(y)} \geq -\frac{(g \circ h)''(y)}{(g \circ h)'(y)} := r_{g \circ h}(y).$$

Then, Theorem 3 in Hammond, John S. III. (1974) implies that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)f(y)dy \implies \int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))f(y)dy. \quad (\text{A.8})$$

Case III: $\bar{a} = 1$ (and $\underline{a} > 0$)

In this case,³⁹

$$\begin{aligned} \int_0^1 h(y) \cdot [f(y) - 1] dy &= \int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy + \int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy \\ &> h(\underline{a}) \cdot \left[\int_0^{\underline{a}} [f(y) - 1] dy + \int_{\underline{a}}^1 [f(y) - 1] dy \right] = 0. \end{aligned} \quad (\text{A.9})$$

The last equality is a consequence of $f(\cdot)$ being a PDF with domain $[0, 1]$. To show that the inequality holds, we note that $h(\cdot)$ is increasing and distinguish two cases. First, let

$$h(\underline{a}) > 0.$$

Then, the inequality in (A.9) holds because

$$\int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy > h(\underline{a}) \cdot \int_0^{\underline{a}} [f(y) - 1] dy$$

and

$$\int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy \geq h(\underline{a}) \cdot \int_{\underline{a}}^1 [f(y) - 1] dy.$$

Second, let

$$h(\underline{a}) = 0.$$

Then, the inequality in (A.9) holds because

$$\int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy = h(\underline{a}) \cdot \int_0^{\underline{a}} [f(y) - 1] dy = 0$$

and

$$\int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy > h(\underline{a}) \cdot \int_{\underline{a}}^1 [f(y) - 1] dy = 0.$$

That is, we have shown that in this case we always have

$$\int_0^1 h(y) dy < \int_0^1 h(y) f(y) dy,$$

i.e., the statement of the lemma trivially holds because the inequalities are never satisfied (with or without globalization). □

³⁹In this case, $F(y)$ first order stochastically dominates $I(y)$. Nevertheless, we cannot directly apply Mas-Colell et al. (1995, Proposition 6.D.1) because we need the inequality in (A.9) to be strict. Theorem 3 in Hammond, John S. III. (1974) could be used for Case III, although no proof is provided in his paper.

A.4 Proof of Theorem 1

Part (i) follows immediately from Lemma 1, which implies that whenever $h(y)$ and $F^{1,m}(y)$ satisfy Condition (9), so will $g(h(y))$ and $F^{1,m}(y)$. We prove part (ii) by means of an example with the desired property. That is, we want to find $h \in \mathcal{H}$, $g \in \mathcal{G}$ and $F^{1,m}(\cdot)$ such that

$$\int_0^1 h(y) \cdot dF^{1,m}(y) > \int_0^1 h(y) dy = \int_0^1 g(h(y)) dy \geq \int_0^1 g(h(y)) \cdot dF^{1,m}(y).$$

Accordingly, let $h(x) = x^{1/2}$ and $g(x) = \frac{4(1-\delta)}{3} \cdot x^2 + \delta \cdot x$, for $\delta > 0$. Clearly, $h \in \mathcal{H}$ and $g \in \mathcal{G}$. It is then a matter of simple algebra to verify that

$$\int_0^1 h(y) dy = \int_0^1 g(h(y)) dy = \frac{2}{3}.$$

Next, consider $F^{1,m}(\cdot)$ defined for every $\varepsilon > 0$ as follows

$$dF^{1,m}(y) = \begin{cases} 1/\varepsilon & \text{if } 0.47 - \varepsilon/2 \leq y \leq 0.47 + \varepsilon/2 \\ 0 & \text{otherwise} \end{cases}.$$

For $\varepsilon > 0$ and $\delta > 0$ arbitrarily low, we obtain

$$\int_0^1 g(h(y)) \cdot dF^{1,m}(y) \approx 0.63 < \frac{2}{3}$$

and

$$\int_0^1 h(y) \cdot dF^{1,m}(y) \approx 0.69 > \frac{2}{3}.$$

Accordingly, $(\mathcal{W}, g \circ h, \bar{\mathcal{F}})$ satisfies PAM, but $(\mathcal{W}, h, \bar{\mathcal{F}})$ does not.

□

A.5 Proof of Theorem 2

The proof of Theorem 2 closely follows the arguments in the proofs of Lemma 1 and Theorem 1 and can be provided upon request. Here, we just show Part (ii) by means of an example with the desired property. That is, we want to find $h \in \mathcal{H}$, $g \in \mathcal{G}$, $F^{0,l}(\cdot)$ and $F^{0,h}(\cdot)$ such that

$$\begin{aligned} \int_0^1 g(h(y)) \cdot \frac{1}{2} (dF^{0,l}(y) + dF^{0,h}(y)) &> \int_0^1 g(h(y)) dy & \text{(A.10)} \\ &= \int_0^1 h(y) dy \geq \int_0^1 h(y) \cdot \frac{1}{2} (dF^{0,l}(y) + dF^{0,h}(y)) . \end{aligned}$$

Take now $h(x) = x^{1/2}$, $g(x) = 2(1 - \delta) \cdot x^4 + \delta \cdot x$, and

$$f^{0,l}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 0 \leq y \leq \epsilon \\ 0 & \text{otherwise} \end{cases},$$

and

$$f^{0,h}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 1 - \epsilon \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

One can easily check that (A.10) holds if we take $\epsilon > 0$ and $\delta > 0$ arbitrarily low.

□

A.6 Proof of Theorem 3

To show the theorem, several cases have to be distinguished. Before we do so, however, we note that under Assumption 4, an equilibrium must exist with and without globalization (see Proposition 2). Hence, for $\underline{\alpha}_g = 1$ and $\bar{\alpha}_h = 0$ the result holds trivially. Moreover, for $\bar{\alpha}_h = 1$ and $\underline{\alpha}_g = 0$, the desired result follows immediately from Theorem 1 and 2, respectively. The latter imply, respectively, that $\bar{\alpha}_g = 1$ and $\underline{\alpha}_h = 0$ in such cases. Henceforth, we can therefore assume $\underline{\alpha}_g \in (0, 1)$ and $\bar{\alpha}_h \in (0, 1)$. We prove both parts of the theorem.

Proof of Part (i): We stress that we are assuming $\underline{\alpha}_g \in (0, 1)$. From Proposition 1, it must then be that

$$\int_0^1 g(h(y))dy = \int_0^1 g(h(y))dF^{\underline{\alpha}_g, m}(y).$$

Then, Assumption 1(iii) and the fact that $g(\cdot)$ is convex imply that

$$\int_0^1 h(y)dy \leq \int_0^1 h(y)dF^{\underline{\alpha}_g, m}(y). \quad (\text{A.11})$$

This follows from Hammond, John S. III. (1974, Theorem 3, Part b.2) along the lines of the proof of Lemma 1. If, on the one hand, (A.11) holds as equality, then by Proposition 1 we know that $\underline{\alpha}_g$ is also an equilibrium without globalization. This immediately implies that $\underline{\alpha}_h \leq \underline{\alpha}_g$. On the other hand, assume that Inequality (A.11) holds strictly. Because of continuity of $\int_0^1 h(y)dF^{\alpha, m}(y)$ in α , there are two options. First,

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{\hat{\alpha}, m}(y)$$

for some $\hat{\alpha} \in (0, \underline{\alpha}_g)$. Using Proposition 1, we obtain that $\hat{\alpha}$ is an equilibrium without globalization, and hence $\underline{\alpha}_h \leq \hat{\alpha} < \underline{\alpha}_g$. Second,

$$\int_0^1 h(y)dy < \int_0^1 h(y)dF^{\alpha, m}(y)$$

for all $\alpha \in (0, \underline{\alpha}_g)$. This is equivalent to saying that

$$\int_0^1 h(y)dy > \int_0^1 h(y) \cdot \left(\frac{1}{2}dF^{\alpha,l}(y) + \frac{1}{2}dF^{\alpha,h}(y) \right) \quad (\text{A.12})$$

for all $\alpha \in (0, \underline{\alpha}_g)$. To derive (A.12) we have used Equation (1). Finally, by continuity of expected payoffs in the matching, Condition (15) must hold and NAM must be an equilibrium without globalization. Hence, $\underline{\alpha}_h = 0 < \underline{\alpha}_g$.

Proof of Part (ii): We stress that we are assuming $\bar{\alpha}_h \in (0, 1)$. From Proposition 1, it must then be that

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{\bar{\alpha}_h,m}(y).$$

Given the above equality, Lemma 1 implies that

$$\int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF^{\bar{\alpha}_h,m}(y). \quad (\text{A.13})$$

As before, Inequality (A.13) allows us to distinguish two scenarios. First, assume that

$$\int_0^1 g(h(y))dy = \int_0^1 g(h(y))dF^{\hat{\alpha},m}(y).$$

for some $\hat{\alpha} \in [\bar{\alpha}_h, 1)$. Then, $\hat{\alpha}$ is an equilibrium with globalization, and hence $\bar{\alpha}_h \leq \hat{\alpha} \leq \bar{\alpha}_g$.

Second, assume that

$$\int_0^1 g(h(y))dy > \int_0^1 g(h(y))dF^{\hat{\alpha},m}(y).$$

for all $\alpha \in [\bar{\alpha}_h, 1)$. Continuity of $\int_0^1 g(h(y))dF^{\alpha,m}(y)$ in α then leads to Condition (9). Proposition 1 then implies that with globalization, PAM is an equilibrium. Hence, $\bar{\alpha}_h < 1 = \bar{\alpha}_g$.

□

Online Appendix

B Mathematical Appendix

In this appendix, we provide further details for some of the discussions in the main text.

B.1 Uniqueness of equilibrium

In this appendix, we discuss uniqueness of the equilibrium. In particular, we show that, given Assumption 4, NAM is the unique equilibrium if and only if

$$V(t^m|\mu(\alpha)) > V(t^m|\mu(0)) \text{ for all } \alpha \in (0, 1], \quad (\text{B.1})$$

while PAM is the unique equilibrium if and only if

$$V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha)) > V(t^l|\mu(1)) + V(t^h|\mu(1)) \text{ for all } \alpha \in [0, 1). \quad (\text{B.2})$$

Condition (B.1) requires that mixed teams expect the lowest payoff when all teams are mixed, while Condition (B.2) requires that the average payoff of a high- and a low-skilled team is lowest when no team is mixed.⁴⁰

To see that NAM (PAM) is the unique equilibrium if and only if Condition (B.1) (Condition (B.2)) is satisfied, note first that in our symmetric setup in which only skills are relevant for ranking, in the case where all teams are mixed, they should all expect to be ranked in any position with equal probability. In fact, from Equation (1), it must be that

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{0,m}(y) = V(t^m|\mu(0)).$$

On the one hand, assume Condition (B.1). Then, for all $\alpha \in (0, 1]$,

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) < V(t^m|\mu(\alpha)) = \int_0^1 h(y)dF^{\alpha,m}(y).$$

This means that there cannot be an equilibrium with $\alpha \in (0, 1]$ —see Proposition 1—, and hence an equilibrium with NAM ($\alpha = 0$) is the only possibility. Conversely, suppose Condition (B.1) does not hold. Then, by Assumption 4, there either must be some $\alpha \in (0, 1]$ such that

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) = V(t^m|\mu(\alpha)) = \int_0^1 h(y)dF^{\alpha,m}(y)$$

⁴⁰There can also be a unique equilibrium with $\alpha \in (0, 1)$. Our main focus is on equilibria with PAM and NAM, respectively, and we thus limit our attention to these cases.

and this α corresponds to an equilibrium—see Proposition 1—, or it must be that

$$\int_0^1 h(y)dy = V(t^m|\mu(0)) > V(t^m|\mu(1)) = \int_0^1 h(y)dF^{1,m}(y),$$

in which case PAM is an equilibrium. In either case, NAM cannot be the only equilibrium.

Next, assume Condition (B.2). Then, it must hold that, for all $\alpha \in [0, 1)$,

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] < \frac{1}{2} \cdot [V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)]. \end{aligned}$$

Accordingly, there cannot be an equilibrium with $\alpha \in [0, 1)$, and an equilibrium with PAM ($\alpha = 1$) is the only possibility. First, the fact that there cannot be an equilibrium with NAM follows immediately from Proposition 1. Second, the fact that there cannot be an equilibrium with $\alpha \in (0, 1)$ follows from the fact that for $\alpha \in (0, 1)$

$$\int_0^1 h(y)dy < \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)]$$

implies that

$$\int_0^1 h(y)dy > \int_0^1 h(y)dF^{\alpha,m}(y)$$

by Equation (1), and from Proposition 1. Finally, suppose that Condition (B.2) does not hold. Then, by Assumption 4, it either must be that for some $\alpha \in [0, 1)$

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] = \frac{1}{2} \cdot [V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)], \end{aligned}$$

and the same reasoning as the one just shown implies that this α must be an equilibrium, or it must be that

$$\begin{aligned} \int_0^1 h(y)dy &= \frac{1}{2} \cdot [V(t^l|\mu(1)) + V(t^h|\mu(1))] > \frac{1}{2} \cdot [V(t^l|\mu(0)) + V(t^h|\mu(0))] \\ &= \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{0,l}(y) + dF^{0,h}(y)], \end{aligned}$$

in which case NAM must be an equilibrium. In either case PAM cannot be the only equilibrium.

B.2 Details on the discussions of Section 6.2

In this appendix, we show how Lemma 1 can be applied to the variant of our model of Section 6.2 to show a result analogous to Theorem 1. We have $\tilde{B}^\alpha(\varphi)$ denote the aggregate distribution of productivities in an economy with matching α

$$\tilde{B}^\alpha(\varphi) := \frac{\alpha}{2}B^l(\varphi) + \frac{\alpha}{2}B^h(\varphi) + (1 - \alpha)B^m(\varphi),$$

and $\tilde{b}^\alpha(\varphi) := \frac{d\tilde{B}^\alpha(\varphi)}{d\varphi}$ the associated PDF. With this notation, we have that positive assortative matching is an equilibrium if and only if

$$\int_{\varphi \in \Phi} \pi^1(\varphi) dB^m(\varphi) \leq \int_{\varphi \in \Phi} \pi^1(\varphi) d\tilde{B}^1(\varphi),$$

which is again a supermodularity condition. Using $y := \tilde{B}^1(\varphi)$, we obtain from integration by substitution

$$\int_0^1 \pi^1(\tilde{B}^{1,-1}(y)) \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))} dy \leq \int_0^1 \pi^1(\tilde{B}^{1,-1}(y)) dy,$$

where, as before, we use a superscript -1 to denote an inverse function and where we have simplified the exposition by assuming that $\tilde{b}^1(\varphi)$ has continuous support. $f^{1,m}(y) := \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))}$ is a PDF on $[0, 1]$ that satisfies

$$\mathcal{A} := \{y \in [0, 1] : f^{1,m}(y) \geq 1\}$$

is a convex and compact set by Assumption 1⁴¹. Moreover, subtracting the constant $\pi^1(\tilde{B}^{1,-1}(0))$ from both sides, we obtain

$$\int_0^1 \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] \frac{b^m(\tilde{B}^{1,-1}(y))}{\tilde{b}^1(\tilde{B}^{1,-1}(y))} dy \leq \int_0^1 \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] dy,$$

where $h(y) := \left[\pi^1(\tilde{B}^{1,-1}(y)) - \pi^1(\tilde{B}^{1,-1}(0)) \right] \in \mathcal{H}$. Hence, we can indeed apply Lemma 1 to generalize Theorem 1.

B.3 Details on the discussions of Section 7.1.2

In this appendix, we show that in case of a patent race as described in Section 7.1.2, the rank distribution of a mixed team satisfies Assumption 1 for any $\lambda^l, \lambda^m, \lambda^h > 0$.

⁴¹Note that

$$\int_0^1 f^{1,m}(y) dy = \int_{\varphi \in \Phi} \frac{b^m(\varphi)}{\tilde{b}^1(\varphi)} \cdot \tilde{b}^1(\varphi) d\varphi = 1.$$

Note first that the rank of a team with skill level k is distributed according to the cumulative distribution $F^{\alpha,k}(y)$, where for all $y \in [0, 1]$,

$$F^{\alpha,k}(y) = \Pr[r^\alpha(z) \leq y] = \Pr[z \geq r^{\alpha,-1}(y)] = 1 - B^k(r^{\alpha,-1}(y)).$$

The second equality follows from $r^{\alpha'}(\cdot) < 0$. Differentiating with respect to y and applying the chain rule, we obtain

$$f^{\alpha,k}(y) = -b^k(r^{\alpha,-1}(y)) \cdot \frac{d}{dy} r^{\alpha,-1}(y) \quad (\text{B.3})$$

$$= -\lambda^k \exp(-\lambda^k r^{\alpha,-1}(y)) \cdot \frac{d}{dy} r^{\alpha,-1}(y). \quad (\text{B.4})$$

Now, we want to show that the set

$$\mathcal{A} := \{y \in [0, 1] : f^{\alpha,m}(y) \geq 1\}$$

is compact and convex. To make progress, note first that this set is equal to

$$\mathcal{A} = \left\{ y \in [0, 1] : \tilde{f}(y) \leq 1 \right\},$$

where

$$\begin{aligned} \tilde{f}(y) &:= \frac{(\alpha/2)f^{\alpha,l}(y) + (\alpha/2)f^{\alpha,h}(y) + (1-\alpha)f^{\alpha,m}(y)}{f^{\alpha,m}(y)} \\ &= \frac{(\alpha/2)\lambda^l \exp(-\lambda^l r^{\alpha,-1}(y)) + (\alpha/2)\lambda^h \exp(-\lambda^h r^{\alpha,-1}(y)) + (1-\alpha)\lambda^m \exp(-\lambda^m r^{\alpha,-1}(y))}{\lambda^m \exp(-\lambda^m r^{\alpha,-1}(y))}. \end{aligned}$$

The equivalence follows from the fact that the numerator in the first row is equal to one by Equation (1). The advantage of expressing the set \mathcal{A} this way is that it allows us to get rid of the term $\frac{d}{dy} r^{\alpha,-1}(y)$ in Equation (B.3). Now, because $r^{\alpha,-1}(y)$ is strictly decreasing, showing that the bounded set $\left\{ y \in [0, 1] : \tilde{f}(y) \leq 1 \right\}$ is closed and convex is equivalent to proving that the set

$$\left\{ z \in [0, \infty) : \tilde{f}(z) \leq 1 \right\}$$

is closed and convex, where $z = r^{\alpha,-1}(y)$. The desired result then follows from noting that

$$\frac{d^2}{(dz)^2} \tilde{f}(z) = \frac{\alpha}{2} \left[\frac{\lambda^l}{\lambda^m} (\lambda^m - \lambda^l)^2 \exp[(\lambda^m - \lambda^l)z] + \frac{\lambda^h}{\lambda^m} (\lambda^m - \lambda^h)^2 \exp[(\lambda^m - \lambda^h)z] \right] > 0.$$

This implies that $\tilde{f}(z)$ can cross one at most once from below.

B.4 Details on the discussions of Section 7.2.2

In this appendix, we show that in the variant of the Melitz (2003)-model considered in Section 7.2.2, conditional on the matching, the mass of entrants is independent of the trade environment.

The productivity of an entrepreneurial team t^k with skill level $k \in \{l, m, h\}$ is drawn from a Pareto distribution with skill-dependent minimum-productivity level

$$b^k(\varphi) = \begin{cases} \frac{\gamma \varphi^{k\gamma}}{\varphi^{\gamma+1}} & \text{if } \varphi \geq \underline{\varphi}^k \\ 0 & \text{otherwise} \end{cases}, \quad (\text{B.5})$$

and where $\underline{\varphi}^l \leq \underline{\varphi}^m \leq \underline{\varphi}^h$. Ignoring knife-edge cases, free entry implies that the (expected) income of the lowest skilled entrepreneurs must equal the wage rate of workers, while higher skilled entrepreneurs earn positive rents. Note that this immediately implies that all high-skilled agents must work as entrepreneurs if some low-skilled agents are to work as entrepreneurs as well.

Now, to simplify the exposition, suppose that entrepreneurial talent is scarce such that there are always some low-skilled teams.⁴² In equilibrium, free entry then implies that the ex-ante expected profit of a low-skilled team has to be equal to twice the wage rate for workers

$$\sum_{j \in \mathcal{I}} \int_{\varphi_{ij}^*}^{\infty} [\pi_{ij}(\varphi) - f_{ij} w_i] b^l(\varphi) d\varphi = 2w_i, \quad (\text{B.6})$$

where \mathcal{I} is the set of countries, f_{ij} are the fixed cost in terms of domestic labor of serving destination country j from country i , w_i is the wage rate in country i , and $\pi_{ij}(\varphi)$ are the variable profits that a firm in country i with productivity φ can make when serving consumers in destination country j . φ_{ij}^* is the well-known productivity cutoff, i.e. a firm in country i with productivity φ_{ij}^* just breaks even when serving destination country j . In equilibrium, a firm in country i will serve destination j if and only if it has productivity $\varphi \geq \varphi_{ij}^*$. The labor market in country i clears if

$$M_{ei} \left[\sum_{j \in \mathcal{I}} \int_{\varphi_{ij}^*}^{\infty} [l_{ij}^v(\varphi) + f_{ij}] b_i(\varphi) d\varphi \right] + M_{ei} f_{ei} = L_i, \quad (\text{B.7})$$

where M_{ei} is the total mass of entering firms, $l_{ij}^v(\varphi)$ the variable labor input for a firm in country i with productivity φ associated with serving destination country j , and f_{ei} is the

⁴²That is, we consider the case where $M_{ei} > \frac{L_i^h}{2}$, where L_i^h denotes the mass of high-skilled labor in country i . This restriction is not essential for the following arguments and it can easily be dispensed with at the expense of additional notational complexity.

fixed cost of labor involved with founding a firm, i.e. $f_{ei} = 2$ if entrepreneurial teams have two team members. $b_i(\varphi)$ is the productivity distribution of all firms in country i , which depends on the matching of entrepreneurs to teams. In particular, let α_i^k , $k \in \{l, m, h\}$, be the share in country i of entrepreneurial teams with skill level k . With this notation, we have that

$$b_i(\varphi) = \alpha_i^l b^l(\varphi) + \alpha_i^m b^m(\varphi) + \alpha_i^h b^h(\varphi) . \quad (\text{B.8})$$

Combining Equations (B.5) to (B.8) and following steps as shown in the online appendix of Melitz and Redding (2014), we get⁴³

$$M_{ei} = \frac{L_i}{f_{ei}} \left[1 + \frac{\sigma(\gamma - 1) + 1}{\sigma - 1} \cdot \frac{\alpha_i^l \underline{\varphi}^{l\gamma} + \alpha_i^m \underline{\varphi}^{m\gamma} + \alpha_i^h \underline{\varphi}^{h\gamma}}{\underline{\varphi}^{l\gamma}} \right]^{-1} . \quad (\text{B.9})$$

Hence, indeed, the mass of entering firms does not directly depend on the trade environment.

⁴³For the case of $\underline{\varphi}^l = \underline{\varphi}^m = \underline{\varphi}^h$, Expression (B.9) reduces to Equation (22) in Melitz and Redding (2014).