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CID Working Paper No. 276
January 2014
Revised Jan. 2019

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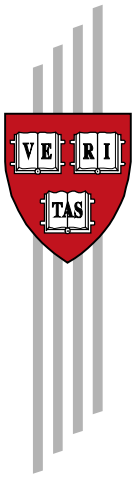

Working Papers

Center for International Development
at Harvard University

Implied Comparative Advantage *

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January 2019

Abstract

The comparative advantage of a location dictates its industrial structure. Current theoretical models based on this principle do not take a stance on how comparative advantages in different industries or locations correlate with each other, or what such patterns of correlation might imply about the underlying process that governs the evolution of comparative advantage. In fact, we find that correlations do appear to exist: industries tend to exhibit output intensities that are systematically correlated across locations, and locations tend to have output intensities that are correlated across industries. We give evidence that these patterns are present in a wide variety of contexts, namely the export of goods (internationally) and the employment, payroll and number of establishments across the industries of subnational regions (in the US, Chile and India). We then calculate the industry intensities that are implied by related industries or related locations, and show that these measures explain much of the location's current industrial structure. Furthermore, the deviations between the actual industry structure and our implied comparative advantage measures tend to be highly predictive of future industry growth, especially at horizons of a decade or more; this explanatory power holds at both the intensive as well as the extensive margin. These results indicate that future productivity is already implied in today's patterns of production.

JEL Codes: O41, O47, O50, F10, F11, F14

*We thank Philippe Aghion, Pol Antràs, Sam Asher, Jesus Felipe, Elhanan Helpman, Asim Khwaja, Paul Novosad, Andrés Rodrìguez-Clare and Dani Rodrik for very useful comments on earlier drafts. We are indebted to Sam Asher and Paul Novosad for sharing the data on India and the Servicio de Impuestos Internos for sharing the data on Chile. All errors are ours.

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David Ricardo (1817)'s seminal theory predicts that locations specialize in the goods in which they have a comparative advantage, meaning that they enjoy a higher relative productivity. Yet these comparative advantages are not random, nor are they set in stone; theories detailing the evolution of a locations' productivities date back to the work of Marshall (1890) more than a century ago. Since then, there have been many studies highlighting the importance of sectoral or regional relatedness in the evolution of comparative advantage or the production structures of regions. Here, we take a complementary stance, proposing that the relatedness of industries and locations could be used to develop a measure of counterfactual or *implied comparative advantage*.

Imagine the following thought experiment. You have conducted the first-ever economic census of your country, containing the output matrix for all cities across all industries. However, due to some accident, your computer randomly erases a few entries in this matrix. How would you guess what those entries were if you had no information other than the surviving part of the matrix? The current theoretical models would not help you use the surviving data to predict the inter-industry variation in output at the city level for the missing data.

In this paper we extend the neo-Ricardian models by assuming that the industrial relatedness causes relative productivities to be differentially correlated across industries in a manner that can be empirically estimated. This structure implies that the comparative advantage of a location in an industry can be estimated from its comparative advantage in related industries, even for the industries that are currently absent from the location. Further, information about the relative productivities of locations with similar attributes should be informative of the relative productivities of industries in a given location. Hence, we can infer the similarity in the productive orientation of locations from the similarity in their output structure. Symmetrically, the intensity with which a location engages in an industry should be related to the intensity with which it engages in similar industries, where industry similarity is calculated from the pattern of coincidence of pairs of industries across all locations. We use these similarity measures to generate predictors of the *implied comparative advantage* of a location in an industry and show that it is strongly predictive of the *revealed comparative advantage* of that country in that industry. In addition, these estimates are strongly predictive of future changes in comparative advantage, whether among industries that already exist in a particular location or among those that have yet to emerge. In terms of our thought experiment, our approach allows us to make estimates of the missing data, and the error terms of our prediction are not just noise, but are actually predictive of future changes.

Given both the national and subnational applicability of our approach, we use tech-

niques or models used in both trade theory and urban and regional economic literature. We base our theoretical models on the international trade literature as many recent regional development studies do (e.g., Davis and Dingel (2014); Costinot et al. (2016); Caliendo et al. (2017)). Our empirical work is more intimately related to related diversification literature (Hidalgo et al., 2018; Boschma, 2017). Our results are important in shaping policy discussions. Especially given the importance of emergence of new industries, we believe that our results could be used in shaping this process as a guide.

Although Ricardo introduced the idea of comparative advantage almost two centuries ago (Ricardo, 1817), the multi-location multi-product version of his model has only recently been formalized and subjected to rigorous empirical testing (Eaton and Kortum, 2002; Costinot et al., 2012). These models infer a location's productivity in a certain industry from its observed pattern of trade and have been successful in explaining a significant portion of bilateral and subnational trade. Yet, these models can only infer the relative productivity of a country in a product if the country actually makes the product but cannot infer the productivity if the country does not. [1] This is an important shortcoming as there are many instances in which it would be useful to infer the productivity level that a country would enjoy in products that it does not currently make. In addition, current Ricardian models assume that the relative productivity parameters across industries are uncorrelated. This implies that the likely productivity of a country in trucks is independent of whether it currently has comparative advantage in cars or in coffee.

Essentially, the Ricardian model can be seen as a reduced form of a more structural model that determines the productivity parameter of the labor inputs. One such model is a factor-based explanation given by Heckscher-Ohlin and later extended by Vanek (1968) where, implicitly, the labor productivity parameter is the consequence of the availability of an unspecified list of other factors of production. These may include many varieties of human capital, geographic factors and technological prowess, among many others. In the Appendix, we show that the essential results and reduced form equations of our approach can be derived from this setting. With a factor-based interpretation, the revealed comparative advantage of a country in a product can be inferred from its revealed comparative advantage in products that have similar production functions or locations that have similar factor endowments. Interestingly, our results can be derived without information regarding production functions or factor endowments.

Our findings do not only pertain to international trade: we obtain similar results when

[1] Deardorff (1984), quoted by Costinot et al. (2012) says that *"If relative labor requirements differ between countries, as they must for the model to explain trade at all, then at most one good will be produced in common by two countries. This in turn means that the differences in labor requirements cannot be observed, since imported goods will almost never be produced in the importing country."*

we use sub-national data on wage bill, employment or the number of establishments for the US, India and Chile. Clearly, a city is an economy that is open to the rest of its country and, hence, the logic behind trade models should be present, albeit with more factor mobility than is usually assumed in trade models. Our results operate both at the intensive and the extensive margins of growth: they correlate with future growth rates of industry-locations, as well as with the appearance and disappearance of new industries in each location.

Related literature

This paper is related to several strands of literature given its scope covers both national and subnational data. Specifically, it builds on Bahar et al. (2014), Hausmann and Klinger (2006, 2007) and Hidalgo et al. (2007) but develops a theoretical framework and explores both the extensive and the intensive margins of industrial evolution of regions. Building of these studies, Boschma and Capone (2015) analyzes the interaction between relatedness and institutions and finds that different varieties of capitalism result in different diversification patterns. Petralia et al. (2017) finds that the related diversification is also important at the technological development of countries especially at initial stages of development. Boschma et al. (2012, 2013) apply a similar approach to understand the regional diversification in Spain. Neffke et al. (2011) show that regions diversify into related industries, using an industry relatedness measure based on the coproduction of products within plants.

These studies could be thought as a part of larger relatedness literature (Hidalgo et al., 2018; Boschma, 2017). Relatedness measures have been used to understand the relationship between technology intensity of an industry and agglomeration (Liang and Goetz, 2018) and to understand how scientific knowledge diffuses between cities (Boschma et al., 2014) as well.

Our results using sub-national data relate to the urban and regional economics literature. For example, Ellison et al. (2010) try to explain patterns of industry co-agglomeration by exploring overlaps in natural advantages, labor supplies, input-output relationships and knowledge spillovers. We do not try to explain co-agglomeration but instead use it to implicitly infer similarity in the requirements of industries or the endowments of locations. Hanlon and Miscio (2017) further show that the historical pattern of location distribution of industries in Britain are shaped by the agglomerative forces as well. Delgado et al. (2010, 2015) and Porter (2003) use US sub-national data to explain employment growth at the city-industry level, using the presence of related industry clusters. Lu et al.

(2016) explores the effect of co-located clusters in the emergence of new clusters and find differential interactions depending on the maturity of the cluster. We show that the observed formation of clusters in a region and the region's implied comparative advantage are intimately linked. Beaudry and Schiffauerova (2009) surveys the literature to determine whether Marshallian forces or diversity of a region is more effective on the economic progress of regions. Our work does not take a stance in that regard but the relatedness measures that we use capture more than the Marshallian forces.

Interestingly, the measures we derive are similar to the collaborative filtering models used in the computer science literature. These models try to infer, for example, a user's preference for an item on Amazon based on their purchases of similar items (Linden et al., 2003), or how they will rate news articles based on the ratings of similar users (Resnick et al., 1994). Here we derive a theoretical rationale for their logic.

Our paper is related to the literature on the Ricardian models of trade (Dornbusch et al., 1977; Eaton and Kortum, 2002; Costinot et al., 2012), where we abandon the assumption of an absence of systematic correlations of relative productivity parameters between industries. For example, Eaton and Kortum (2002) assumed that the productivity parameters are drawn from a Frechét distribution, except for a common national productivity parameter. Costinot et al. (2012) relaxed this assumption by assuming a country-industry parameter, but no correlation across industries in the same country. These assumptions are clearly rejected by the data, as there is very significant correlation across industries in the same country. In our results, we show that there is a systematic correlation in the patterns of comparative advantage across pairs of industries across all countries. We also show that there is a systematic correlation of the patterns of comparative advantage between pairs of countries across all industries. We assume instead that technological relatedness across industries causes relative productivities to be correlated.

The patterns we observe in the data allow us to derive implied comparative advantage estimates. It has the advantage of being able to estimate relative productivities for industries that have zero output. Moreover, the implied parameters estimated are strongly correlated with future relative productivities implying that they capture something more fundamental than the relative productivities that are calculated from contemporaneous trade. Previous Ricardian literature, however, cannot infer relative productivities of industries that do not exist. An exception is Costinot et al. (2016) where they estimate implied or counter-factual productivity parameters for agricultural industries using agronomic models and data. This approach requires a detailed knowledge of agricultural production functions and hence cannot be easily extended to other industries. Our approach can be extended to all industries.

This paper is structured as follows. Section 1 derives our predictors using a modified Ricardian framework. Section 2 discusses the data. Section 3 presents our results for the intensive margin. Section 4 discusses our results on the growth of industries in location. Section 5 contains our results for the extensive margin. In Section 6 we conclude with a discussion of the implications of our findings.

1 Theoretical motivation

In this section, we use a modified Ricardian framework to show how the similarities between the output of industries across different locations (and similarities between the output of locations across different industries) can contain information on the "true" comparative advantage of a location, i.e. the hidden match between the requirements of industries and the ability of locations to meet those requirements. As we argue in the Introduction, a Ricardian model of trade that assumes that the productivity parameter of a country in an industry is a random realization from a probability distribution would not be able to explain the patterns of co-location of industries in countries, or the co-location patterns of the same industry across countries. However, it is possible to make a Ricardian model compatible with these observations by incorporating the assumptions that products differ in their technological relatedness, and that countries tend to have similar productivities in technologically-related products. With this assumption, we can motivate our results in a Ricardian framework as stating that a country will export a product with an intensity that is similar to that with which countries with similar patterns of comparative advantage export that product. By the same token, it would also export that product with an intensity that is similar to that with which it exports technologically related products. In the Appendix, we also derive measures that capture the similarity between industries and between locations using a factor-based model like Heckscher-Ohlin-Vanek (Vanek, 1968) model.

Here, we will construct a particular relation between the requirements of an industry and endowments of a location, showing how their interaction might effect the observed global patterns of output, and what can be inferred from these patterns. Let's denote the size (e.g., output, payroll, employment, or number of establishments) of an industry i in a location l with y_{il}. Suppose the total industry size in the sample universe (which is the world if we are dealing with a national data or the country if we are dealing with the subnational case) is $Y_i \equiv \sum_l y_{il}$. The location's expected size will depend on the size of the location relative to size of the sample universe. Let's denote size of the location with

W_l and size of the sample universe with $W \equiv \sum_l W_l$. With this in hand, we can write the expected size of the location as:

$$\hat{y}_{il} = Y_i \frac{W_l}{W}.$$

We can develop a measure of comparative advantage by dividing the observed size of the industry to its expected size:

$$r_{il} = \frac{y_{il}}{\hat{y}_{il}}.$$

By taking logs, using the definition of the expected size and re-arranging the terms we arrive at:

$$\log(y_{il}) = \log(r_{il}) + W - \log(Y_i) - \log(W_l) \tag{1.1}$$

In particular, if y_{il} is the exports of country l in industry i, and W_l is the total exports of the country r_{il} becomes Balassa (1964)'s Revealed Comparative Advantage (RCA) measure. Instead, if we use W_l to be the population of the location, we arrive at the Revealed per-Capita Advantage (RpCA), which will be more explicitly defined in Section 2.2. On the other hand, if we use y_{il} to be the number of employees in industry i in location l and W_l as the total employment in location l we arrive at the Location Quotient (LQ) measure that has been widely used. We can also use y_{il} to be the total payroll in industry i in location l, and W_l as the total payroll in location l and come up with a new measure of comparative advantage.

In a sense, Equation 1.1 is a decomposition of the size of an industry in a location. It has a component that captures the dynamics in the total size of the industry (Y_i), another component that captures the location specific dynamics (W_l), and a portion that is location-industry specific (r_{il}). In our empirical analysis, we will be focusing in this interaction term. Normalizing output values in this way is attractive: it lets us strip out the scaling effects that exist purely at the location level (e.g. the population size of a country) and the industry level (e.g. the global demand for a commodity), and instead focus on explaining the interplay between industries and locations. That is, instead of asking questions like "Why is employment growth higher in Boston than in Kansas City?" or "Why is employment in retail services growing faster than electronics manufacturing?" we ask questions in the class of "Why is electronics manufacturing growing relatively faster in Boston than in Kansas City?"

Having defined our measure of industry-location intensity, we can now build a Ricardian framework to model how these intensities are generated through the interaction of industry requirements and location endowments. We will assume that the efficiency with which industry i functions in location l depends on the distance between the requirements

of industry i and endowments of location l. Suppose the requirements of the industry i are characterized by a parameter ψ_i, which is a number on the real circle with a circumference of 1, which we denote by $\mathbb{U}$. [2] The endowments of location l is characterized by a parameter λ_l, also on $\mathbb{U}$. The output intensity of industry i in location l will depend on the congruity between the requirements of the industry, ψ_i, and the endowments of the location, λ_l. More concretely,

$$r_{il} = f\left(d(\psi_i, \lambda_l)\right) \tag{1.2}$$

where d is the distance on the unit circle $\mathbb{U}$, and f is a strictly decreasing function of that distance, such that $f(0) = 1$ and $f(0.5) = 0$. As can be observed, output intensity will be maximized when $\psi_i = \lambda_l$; in the opposite case, where ψ_i and λ_l are on antipodal sides of the circle (and distance is 0.5), output would be zero. In reality, we would not be able to observe $\psi_i$and λ_l directly, but we can measure r_{il}. The basic intuition is that information about ψ_i and λ_l is contained in the presence of other industries in the same location or the presence of the same industry in other locations. For example, the difference between a location's comparative advantage in two industries, i and i', is an increasing function of the distance between the ψ_i and $\psi_{i'}$. By the same token, the difference in the intensity of output of the same industry across two locations l and l' would be an increasing function of the difference in the λ_l and $\lambda_{l'}$.

We can generalize this intuition by taking advantage of the information contained in the share of output of all industries in all locations. Suppose we start with the normalized output intensity r_{il} for each industry in each location. We can calculate a matrix that contains correlations of each industry pair across all locations. We define as the product space similarity matrix $\phi_{ii'}$ between two industries i and i' as the scaled Pearson correlation between r_i and $r_{i'}$ across all locations:

$$\phi_{ii'} = (1 + \text{corr}\{r_i, r_{i'}\})/2 \tag{1.3}$$

Symmetrically, we define the country space proximity matrix $\phi_{ll'}$ between two locations l and l' as the scaled Pearson correlation between r_l and $r_{l'}$ across all industries:

$$\phi_{ll'} = (1 + \text{corr}\{r_l, r_{l'}\})/2 \tag{1.4}$$

If we assume that $\psi_i$and λ_l are uniformly distributed on the unit circle, and if we use a

[2] We chose the unit circle to avoid boundary effects of the space. For instance, for an interval like $[0, 1]$, the boundaries, 0 and 1, will introduce break points. In reality the technological space is multi-dimensional but here we introduce a one-dimensional version to illustrate our results. Our results are not sensitive to choice of the technological space.

specific productivity function, $f\left(d(\psi_i, \lambda_l)\right) = 1 - 4d^2(\psi_i, \lambda_l)$, then we can derive a closed form expression for the expected value of the $\phi_{ii'}$ as a monotonic function of the distance between the ψ_is (see Appendix for the details of the calculation):

$$\phi_{ii'} = 1 - 15\left(d(\psi_i, \psi_{i'}) - d^2(\psi_i, \psi_{i'})\right)^2 \tag{1.5}$$

Similarly, our location-location similarity $\phi_{ll'}$ is a monotonic function of the distance between the endowment parameters λ_l and $\lambda_{l'}$:

$$\phi_{ll'} = 1 - 15\left(d(\lambda_l, \lambda_{l'}) - d^2(\lambda_l, \lambda_{l'})\right)^2 \tag{1.6}$$

Note that for distance $d = 0$, the expected proximity would be 1. If distance is equal to its maximum value ($d = 1/2$) then the expected proximity would be the minimum.

We conclude that these two matrices carry information about the similarity in the requirements of pairs of industries and the endowments of pairs of locations. Thus our proximity measures use the information contained in the industry-location matrix to relate the technological requirements of an industry with the endowments of a location.

1.1 Calculating the implied comparative advantage

Equipped with our industry similarity and location similarity metrics, we can now develop a metric for the implied comparative advantage of an industry in a location. Assume that we do not observe r_{il}, the intensity of industry i in location l. However, suppose we observe a second industry, i', which appears in extremely similar intensities to industry i in the same locations, meaning that $\phi_{ii'} \approx 1$. Likewise, suppose we observe a second location, l', which tends to have extremely similar levels of intensity as l across all industries, meaning that $\phi_{ll'} \approx 1$. Based on equations 1.5 and 1.6, we know that

$$\phi_{ii'} \approx 1 \Rightarrow \psi_i \approx \psi_{i'}$$

and

$$\phi_{ll'} \approx 1 \Rightarrow \psi_l \approx \psi_{l'}$$

Plugging these into our formula for r_{il} would imply that $r_{i'l} \approx r_{il}$ and $r_{il'} \approx r_{il}$. That is, even if we do not observe the intensity of industry i in location l, we can proxy it based on the intensity of a highly similar industry in the same location, or based on the intensity of the same industry in a highly similar location.

In the real world, however, it is rare to have nearly identical pairs of industries or

locations; most have no perfect comparator.[3] Thus, even if we base our proxy on the single most related industry, we may be introducing a large error whenever $\phi_{ii'} < 1$ or $\phi_{ll'} < 1$. In addition, there might be severe measurement errors in our outputs. If we instead average over a subset of the most similar industries or locations, and weigh our results by the degree of similarity, we may "average out" some of these errors. Following this logic, our expected value of the r_{il} would be the weighted average of the intensity of the k nearest neighbors $r_{i'l}$ (Sarwar et al., 2001) where both the "nearness" and the weights are given by the similarity parameters $\phi_{ii'}$. We refer to this variable proxying for the implied comparative advantage as the product space density:

$$\widehat{r}_{il}^{[I]} = \sum_{i' \in I_{ik}} \frac{\phi_{ii'}}{\sum_{i'' \in I_{ik}} \phi_{ii''}} r_{i'l} \tag{1.7}$$

where I_{ik} is the k nearest neighbors of industry i:

$$I_{ik} = \{i' | Rank\left(\phi_{ii'}\right) \leq k\} \tag{1.8}$$

We can also build a similar metric using the location similarity indices. With this, the implied comparative advantage of an industry in a location would be the weighted average of the intensity of that industry in the k most related locations:

$$\widehat{r}_{il}^{[L]} = \sum_{l' \in L_{lk}} \frac{\phi_{ll'}}{\sum_{l'' \in L_{lk}} \phi_{ll''}} r_{il'} \tag{1.9}$$

with the set L_{lk} defined as:

$$L_{lk} = \{l' | Rank\left(\phi_{ll'}\right) \leq k\} \tag{1.10}$$

We refer to this variable as the country space density. We will explore the degree to which the product space and country space densities can predict the actual value of the location-industry cells using a toy model where we exactly know all the underlying parameters.

1.2 Simulating the estimators on a toy model

At this point, we have shown that one can construct an implied comparative advantage proxy for a given industry and location. But what is the value of such a proxy? After all, this information is not useful if the output patterns we observe already reflect the true comparative advantage of each location. However, we could also imagine a world

[3] See section 2.2 below for more on the distribution of the similarity values.

in which there is a gap between a location's true comparative advantage and its observed comparative advantage. We can capture this in our model by incorporating an additive random error:

$$\tilde{r}_{il} = r_{il} + \varepsilon_{il}, \qquad \varepsilon_{il} \sim N(0, s\sigma^2) \tag{1.11}$$

Here, the"true" output of an industry, r_{il}, is determined as before, solely by the distance between the technological requirement of the industry ψ_i and the technological ability of the location λ_l. We will thus now call r_{il} the long-run equilibrium output. Let us assume that the output of each industry-location can deviate from this equilibrium value because of a disturbance term ε_{il}. We model ε_{il} as normally distributed, and set its variance equal to the variance of r_{il} times a parameter s, the *noise-to-signal ratio*, which we will vary in our simulations. As a result of these assumptions, we no longer observe the equilibrium output r_{il} but instead observe only the current output, $\tilde{r}_{il}$. We then construct the similarity indices and densities as before, but using the observed $\tilde{r}_{il}$ values as inputs; since the error term is randomly distributed (and may cancel out), the densities may be able to capture information on the underlying equilibrium output by looking at similar industry or location pairs. This raises the question we want to test: is our measure of implied comparative advantage a better predictor of the long-run equilibrium output than the observed comparative advantage?

We can now build our toy model to answer this question. We set the dimensions of the model to $N_i = 100$ industries by $N_l = 100$ locations, and assume a uniform distribution of the ψ_i and the λ_l along the unit circle $\mathbb{U}$. We then use these parameters to calculate the Y_{il} values. For the error term ε_{il}, we want to see what happens as we increase its size relative to Y_{il}. Thus, we set s to a range of six different values, going from 0 (no error) to 1 (equal parts signal and noise) to 4 (four times more noise than signal). We find that the average standard deviation of r_{il} converges to 0.298, so we use that value for σ^2. Next, we build the product space and country space densities using the $\tilde{r}_{il}$ values. Following Duda et al. (2012) we set the k parameter to $k = \sqrt{N_i} = \sqrt{N_l} = 10$; that is, densities are built using the 10 most similar industries or locations.[4] Finally, we measure the predictive power of the PS and CS densities (and the mean of the two) by Pearson correlating them with the equilibrium output intensities; we can benchmark the strength of these correlations against similar correlations between the (noisy) observed and equilibrium output values.

Table 1 gives the R^2 statistics from these correlations, averaged over 5,000 simulations. The PS and CS densities perform quite well at all error levels; the R^2 values are quite high (0.962-0.991) when the error term is smaller in magnitude than the "true" output intensity

[4]However, further simulations suggest that a wide range of k values yields similar results; likewise, our empirical results are robust to changes in k.

Table 1: Mean R^2 of correlations with equilibrium output, for 5,000 simulations

	Mean R^2 of correlations between equilibrium output and			
Noise-to-signal ratio (s)	Output	PS Density	CS Density	Mean of PS & CS densities
0%	1.000	0.991	0.991	0.995
25%	0.941	0.984	0.984	0.990
50%	0.800	0.963	0.963	0.977
100%	0.500	0.880	0.880	0.923
200%	0.200	0.608	0.608	0.700
400%	0.059	0.124	0.124	0.166

component. The densities also hold hold well as the size of the error term grows, with R^2 only decreasing greatly at the highest noise level tested (4:1 noise to signal ratio). These values are also highly consistent, with standard errors all less then a tenth of a percentage point.

Most importantly, the density indices also perform well compared to the observed output intensity (the $\tilde{r}_{il}$ values). As expected based on their construction, $\tilde{r}_{il}$ values also correlate with the underlying equilibrium output (the r_{il} values); when the error term is nonexistent, observed output intensity is identical to equilibrium output intensity. However, as we increase the ratio of noise to signal (s), observed intensity becomes an increasingly weak correlate of equilibrium output. The explanatory power of the PS and CS densities also decreases with increasing noise, but at a much slower rate; at s = 100%, the densities are still strongly associated with the equilibrium output (R^2= 0.88). This confirms our prediction: in a "noisy" world, where industry-locations are far from their equilibrium output, the implied output intensities may be a better predictor of the equilibrium than the observed output intensities, since some of the noise will average out.

Finally, note that the explanatory power of the PS and CS densities are virtually identical; this is expected, since their formulas are mirror images of each other (and since we set the number of locations and industries to be the same). Note also that the mean of the two densities is a slightly stronger predictor than either one individually; this suggests that there is some information in each that is not captured by the other.

1.3 Hypotheses

Before beginning our empirical investigation, we can use these outcomes from our theoretical model and simulations to set some hypotheses:
H1: The implied comparative advantage measures (CS and PS density) should be strongly correlated with current output intensities.
H2: The implied comparative advantage measures contain information on the long-run equilibrium output intensities of industry-locations; as such, the gap between implied comparative advantage and observed comparative advantage (i.e. the regression residuals) will be correlated with industry-location growth over long periods.
H3: The implied comparative advantage can be calculated for even the products that a location does not currently make. Therefore, the implied comparative advantage is predictive of which industries will emerge in a country.
H4: From the simulation results, the PS and CS density variables will have some non-overlapping information, meaning that we expect higher explanatory power from regressions which include them both.

2 Data and Methods

We now turn to the application of our approach to real data using both international and subnational datasets, which cover different countries, time periods and economic variables. After constructing our density indices, we separate our analysis between the exploration of the intensive and extensive margins. We first study the growth rates of industry-location cells, which can only be defined for cells that start with a nonzero output. Later, we study the extensive margin by looking at the appearance of industries that were not initially hosted in a particular location. For each analysis, we fit the density variables for the implied comparative advantage to current output levels, and then conduct out-of-sample regressions to explain either output growth or the appearance and disappearance of industries.

2.1 Data

We begin by using trade data to study the industry-location relationship at an international scale. Here we use UN COMTRADE data, cleaned according to the process detailed in Bustos and Yildirim (2019). Exports are disaggregated into product categories according the Harmonized System four-digit classification (HS4), for the years 1995-2016

(earlier years are available, but use a different product classification system, and might introduce error due to major continuity breaks when converting). We restrict our sample to countries with population greater than 1.2 million and total exports of at least $1 billion in 2008. We also remove Iraq (which has severe quality issues), Serbia-Montenegro (which split into two countries during the period studied), and Namibia and Botswana (which lack customs data for the initial five years of the period). We drop two products, "Natural cryolite or chiolite" (HS4 code 2527), and "Vegetable materials used for brooms" (HS 1403) as their world trade both fall to zero in the mid-2000s; we also exclude the miscellaneous code HS 9999 ("Commodities not specified according to kind"). These restrictions reduce the sample to 122 countries and 1240 products that account for 94.4% of world trade and 93.3% of the world population in 2016.

In addition to the international trade data, we test our model on three national datasets that quantify the presence of industries in subnational locations. We use the US Census County Business Patterns (CBP) database from 2003-2011. It includes data on employment and number of establishments by county, which we aggregate into 708 commuting zones (CZ; Tolbert and Sizer (1996)), and 1,086 industries (NAICS 6-digit). This dataset also provides annual payroll data for 698 CZ and 941 NAICS industries.[5] Our Chile dataset comes from the Chilean tax authority, *Servicio de Impuestos Internos*, and includes the number of establishments based on tax residency for 334 municipalities and 681 industries, from 2005 to 2008 (Bustos et al., 2012). Lastly, we study India's economic structure using the Economic Census, containing data on employment for 371 "super-districts" and 209 industries, for the years 1990, 1998 and 2005.[6] For all the datasets above, we include only industries and regions that have non-zero totals for each year (as we do with the international export data). This approach effectively removes discontinued or obsolete categories.

2.2 Constructing the model variables

First, we build the similarity and density indices for the implied comparative advantage introduced above for each dataset. Our first step is to normalize the export, employment and payroll data to focus on the intensity of each industry-location link (as discussed above), and to facilitate comparison across location, industry and time. We use the ex-

[5]The discrepancy between employment and establishment versus payroll sample sizes comes from the data suppression methods of Census Bureau. To protect the privacy of smaller establishments, the CBP occasionally discloses only the range of employment of an industry in a location, e.g., 1 to 20 employees. In these censored cases, we use the range's midpoint as the employment figure (see Glaeser et al. (1992)). However, the CBP offers no payroll information in these cases, leaving a smaller payroll sample.

[6]This dataset was constructed by Sam Asher and Paul Novosad, who kindly shared it with us.

ports per capita as a share of the global average in that industry. This can be seen as a variant of Balassa's revealed comparative advantage (RCA) index (Balassa, 1964), but using the population of a location as a measure its size rather than its total production or exports (Bustos et al., 2012) This change is valuable because it eliminates the impact of the movement in output or prices of one industry on the values of other industries. However, our results are robust to the use of standard RCA instead of RpCA ? see Appendix. We formally define the Revealed per-Capita Advantage (RpCA) of location l in industry i in year t_0 as:

$$R_{il,t_0} = \frac{y_{il,t_0} / pop_{l,t_0}}{\sum_l y_{il,t_0} / \sum_l pop_{l,t_0}} \tag{2.1}$$

where y_{il,t_0} is the export, employment or payroll value, pop_l is the population in location l, and t_0 is the base year. Note that locations with very low populations will tend to have higher R_{il} values. To address the potential bias against high-population locations, we cap R_{il} at $R_{max} = 5$ when building our similarity indices (Equations 2.2 and 2.3 below). [7] We do not normalize the data for the number of establishments.

At this point, we can use the normalized industry intensity values, R_{il}, to build the similarity indices defined above:

$$\phi_{ii'} = (1 + \text{corr}\{R_i, R_{i'}\})/2 \tag{2.2}$$

$$\phi_{ll'} = (1 + \text{corr}\{R_l, R_{l'}\})/2 \tag{2.3}$$

In other words, two industries are similar if different locations tend to have them in similar intensities. Likewise, two locations are similar if they tend to harbor the same industries with a similar intensity. Though we use the Pearson correlation here, we obtain comparable results using other similarity measures, namely cosine distance, Euclidean distance, the Jaccard index, minimum conditional probability (Hidalgo et al., 2007) and the Ellison-Glaeser co-agglomeration index (Ellison and Glaeser, 1999).

Tables 2 and 3 show the top ten most similar pairs of countries and products in 2010. We note that the most similar are countries in close geographic proximity, a phenomenon that can be explained by geological and climate effects as well as regional knowledge spillovers (Bahar et al., 2012). The list of most similar pairs of products is dominated by machinery products, especially those in the "Boilers, Machinery and Nuclear Reactors," category (HS2 code 84). This matches the observation in Hausmann et al. (2011) that the

[7] We specifically set the ceiling at $R_{max} = 5$ because this is the highest possible RpCA value for the most populous country in the world, China. In a hypothetical industry i where China exports the entire industry's output, then $R_{i,China} = pop_{World} / pop_{China} \approx 5$ on average over the period studied.

machinery-related industries are highly interconnected.

Table 2: Most similar location pairs, international trade, 2010

Location l		Location l'		Location Similarity
COD	Congo, DR	COG	Congo	0.8081
CIV	Côte d'Ivoire	CMR	Cameroon	0.7987
CIV	Côte d'Ivoire	GHA	Ghana	0.7844
SWE	Sweden	FIN	Finland	0.7640
KOR	South Korea	JPN	Japan	0.7631
SDN	Sudan	ETH	Ethiopia	0.7622
KHM	Cambodia	BGD	Bangladesh	0.7543
LTU	Lithuania	LVA	Latvia	0.7526
GHA	Ghana	CMR	Cameroon	0.7519
DEU	Germany	AUT	Austria	0.7499

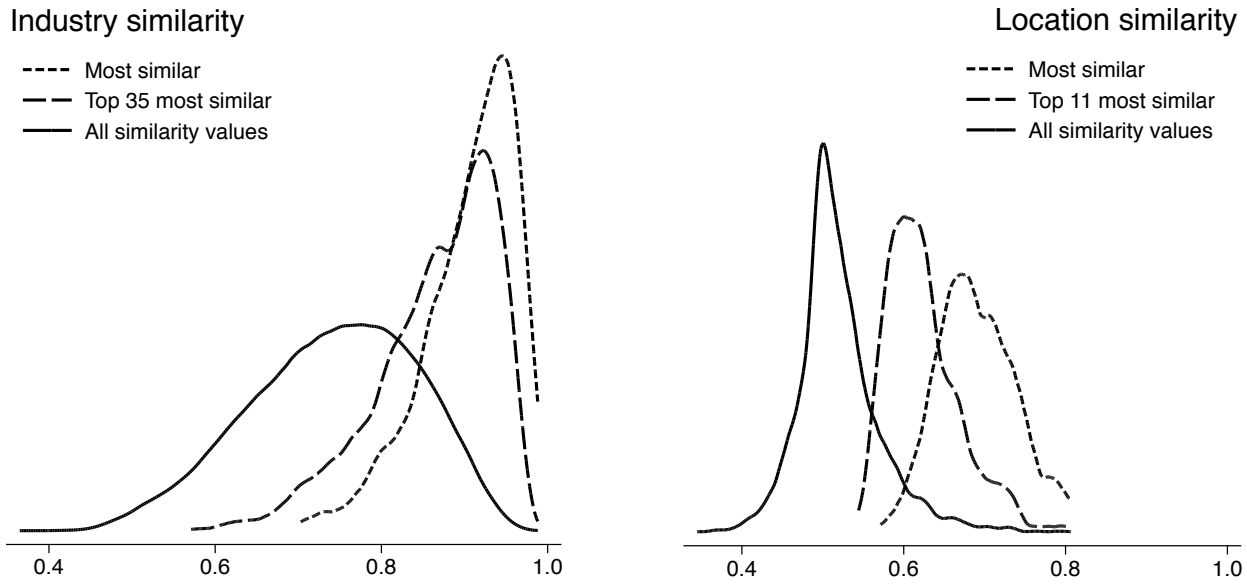
Table 3: Most similar industry pairs, international trade, 2010

Industry i		Industry i'		Industry Similarity
8481	Valves	8413	Liquid Pumps	0.9808
8409	Engine Parts	8483	Transmissions	0.9808
8485	Boat Propellers	8484	Gaskets	0.9784
8481	Valves	8409	Engine Parts	0.9754
7616	Aluminium Products	7326	Iron Products	0.9752
8481	Valves	8208	Cutting Blades	0.9747
8483	Transmissions	8413	Liquid Pumps	0.9747
8413	Liquid Pumps	8409	Engine Parts	0.9745
8208	Cutting Blades	8207	Interchangeable Tools	0.9743
8503	Electric Motor Parts	7326	Other Iron Products	0.9740

Figure 1 show the full distribution of the similarity values; also shown are the subsets of the k most similar industries or locations, where k is set to the square root of the total number of industries ($\sqrt{N_i}$) or locations ($\sqrt{N_l}$). In Figure 1 left, we see that the overall distribution of the industry similarity values is quite broad, with most values in the 0.6 to 0.9 range. However, if we focus on each industry's single most similar comparator industry, then the values are much greater and tighter, mainly rising above 0.9. Values are similarly high when we extend the scope to each industry's 35 most similar comparators. Figure 1 right shows the distribution of the location similarity values. Compared to the industry similarity values, these values appear much lower overall, with the distribution

peaking at around 0.5 (which corresponds to a correlation of zero).[8] However, we see much improvement by limiting the scope to most similar location pairs, and (to a lesser degree) the top 11 most similar locations. For both industries and locations, this illustrates our motivation for including the nearest neighbor filters to remove poor comparators from consideration.

Figure 1: Distribution of similarity values.



Having built our similarity indices, we can use them to recreate our density indices from equations 1.7 and 1.9, replacing the r_{il,t_0} with R_{il,t_0}:

$$w(u)_{il}^{[PS]} = \sum_{i' \in I_{iu}} \frac{\phi_{ii'}}{\sum_{i'' \in I_{iu}} \phi_{ii''}} R_{i'l,t_0} \tag{2.4}$$

where I_{iu} is the u nearest neighbors of industry i. Similarly

$$w(v)_{il}^{[CS]} = \sum_{l' \in L_{lv}} \frac{\phi_{ll'}}{\sum_{l'' \in L_{lv}} \phi_{ll''}} R_{i'l,t_0} \tag{2.5}$$

with the set L_{lv} is is the v nearest neighbors of location l. As before, we set the neighborhood size u for product space to $\sqrt{N_i} \approx 35$ and v to $\sqrt{N_l} \approx 11$ nearest neighbors following

[8]Note also that there are no cases in which industry or location similarity is close to 0: the minimum is 0.345, the similarity between the US and China's export intensities. If there were in fact many similarity values near 0, then it would be possible to build an implied comparative advantage measure using the most "perfectly dissimilar" pairs. Instead, the pairs seem to range from highly similar (at best) to unrelated (at worst).

Duda et al. (2012). These indices serve as our proxies for a location's implied comparative advantage in an industry.

3 Estimating the initial industry-location cells from the values of all other industry-location cells

As argued above, the density variables derived above are the expected value of the output intensity of any cell, given the values of other cells. To see how well they fit, we estimate the following equation:

$$\log(R_{il,t_0}) = \alpha + \beta_{PS} \log\left(w(u)_{il}^{[PS]}\right) + \beta_{CS} \log\left(w(v)_{il}^{[CS]}\right) + \varepsilon_{il,t_0} \tag{3.1}$$

where ε_{il,t_0} is the residual term.

Table 4: OLS regression of international exports by industry-location, 1995

	(1)	(2)	(3)
	Exports, 1995 (Revealed Comparative Advantage, log)		
Product Space Density (log), 1995	0.962***		0.765***
	(0.013)		(0.019)
Country Space Density (log), 1995		0.965***	0.292***
		(0.038)	(0.021)
Adjusted R^2	0.627	0.485	0.645

$N = 93,979$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4 shows that both the product space density and the country space density terms are highly significant ($p < 0.01$), with coefficients very close to unity. As expected, the terms also explain a very large fraction of the variance of the country-product export intensity, though the product space density generates a significantly higher R^2 than the country space density. Together, they explain nearly two thirds of the variation in export intensity. Table 5 shows the regressions for the US, India and Chile datasets. In all, both product space density and country space density are significant ($p < 0.01$ in all cases), and yield coefficients that sum to 0.990 on average (from 0.81 to 1.29). As with the export data, R^2 values are substantial, especially for the establishment count datasets. This suggests

that the value of an industry-location cell in our data can be estimated with some accuracy based on the values of other industry-location cells in the matrix. However, as with any estimation, errors are made. Are these errors just noise, or do they carry information about the evolution of the system? We turn to this question in the next section.

Table 5: OLS regression of initial employment, payroll and establishments by industry-location.

	(1)	(2)	(3)	(4)	(5)
	USA, 2003 employees	USA, 2003 payroll	India, 1990 employees	USA, 2003 establishments	Chile, 2005 establishments
	Revealed comparative advantage (log)			log	log
Product Space density (log)	0.620*** (0.010)	0.476*** (0.016)	0.530*** (0.018)	0.293*** (0.021)	0.306*** (0.015)
Country Space density (log)	0.556*** (0.010)	0.363*** (0.018)	0.759*** (0.013)	0.517*** (0.018)	0.531*** (0.011)
Observations	279,439	89,378	49,651	278,946	50,373
Adjusted R^2	0.267	0.355	0.376	0.787	0.601

Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

4 Growth regressions

The fact that our density variables for the implied comparative advantage can explain current locational intensities is interesting, but more surprising is the fact that the residual is informative of future industry-location growth. Formally, we test for this by regressing the growth rate of the industry-location pair on the residuals that we obtain from the first stage introduced in the previous section. [9] We construct the variables as follows. We use the standard definition of the annualized growth rate of y_{il}:

$$\dot{y}_{il} = \frac{1}{t_1 - t_0} \log \left(y_{il,t_1} / y_{il,t_0} \right) \tag{4.1}$$

where t_0 and t_1 are the initial year and final year, respectively. However, there are a large number of locations with initial output of zero for which we cannot define a growth rate. Likewise, cases in which final output is zero (i.e., $y_{il,t_1} = 0$) are also problematic because it introduces a hard boundary that would bias the estimates. We manage these

[9] Instead of the residual, if we put the implied comparative advantage value, our results improve. But here, we want to show that even the residual term is predictive of the future growth.

issues by separately analyzing the intensive and extensive margins. In this section, we examine growth in the intensive margin by restricting our regression sample of industry-locations to those in which $y_{il,t_0} \neq 0$ and $y_{il,t_1} \neq 0$. In Section 6, we use a *probit* regression model to examine the probability of industry appearance (i.e., growth from zero) and disappearance (i.e., collapse into zero).

Our growth regression takes the following form:

$$\dot{y}_{il} = \alpha + \beta_\varepsilon \varepsilon_{il,t_0} + \gamma c_l + \delta d_i + e_{il} \tag{4.2}$$

where α is the constant, β_ε is the regression coefficient on the residual, γ and δ are the coefficients on location and industry control variables and e_{il} is the error term of the regression.

Table 6 shows a set of growth regressions using our international export data. The dependent variable is the growth rate in the industry-location cell. The first three columns in Table 4 use as independent variable the error terms from the three regressions in Table 6. They show that the residual using both product space and country space densities, as well as both of them combined are highly significant predictors of growth and explain between 15 and 18 percent of the variance of growth between 1995 and 2016. The residual terms for PS and CS density explain roughly equal parts of the variance; as before, the highest R^2 value comes from both terms together, suggesting that their residuals also contain non-overlapping information. Both terms have the expected negative sign, and are significant at $p < 0.01$.

We now look at the robustness of these equations with respect to the inclusion of other relevant industry and location variables. First we include some basic controls regarding the initial global size of the industry in question as well as the total initial exports and population of the location; these correspond with the industry-level and location-level components of the decomposition in equation 1.1. Note that these variables are constructed using information from the base year of the regression alone. Column 4 shows that these variables, on their own, are significantly related to subsequent growth, as noted by Glaeser et al. (1992); however, Column 5 indicates that they do not substantially affect the magnitude and significance of the density residuals, and instead see their own significance decrease.

Table 6: OLS regression of export growth of an industry in a country (1995-2016)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Growth in exports (log), 1995-2016								
Residual, Product	-0.023***		-0.013***		-0.012***		-0.019***		-0.018***
Space density, 1995	(0.001)		(0.002)		(0.002)		(0.002)		(0.001)
Residual, Country		-0.020***	-0.012***		-0.009***		-0.007***		-0.009***
Space density, 1995		(0.001)	(0.002)		(0.002)		(0.001)		(0.001)
Industry-location				-0.012***	-0.003**	-0.011***	0.001	-0.021***	-0.000
exports, 1995 (log)				(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Location population				0.012***	0.005	0.011***	0.002		
1995 (log)				(0.003)	(0.003)	(0.002)	(0.002)		
Global industry total				0.014***	0.006***	0.011***	0.001		
1995 (log)				(0.001)	(0.001)	(0.001)	(0.001)		
Radial industry growth						0.928***	0.969***		
1995-2016 (log)						(0.019)	(0.017)		
Radial location growth						0.748***	0.944***		
1995-2016 (log)						(0.112)	(0.114)		
Industry FE								Yes	Yes
Location FE								Yes	Yes
Adjusted R^2	0.151	0.154	0.178	0.121	0.184	0.234	0.325	0.411	0.439

$N = 93,979$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Next, we introduce controls that account for information beyond the base year, namely the overall rate of growth for each location and each industry; we refer to these controls as the *radial growth* variables. We show that the information captured by the error term is orthogonal to radial growth of the industry and the location. To express radial industry growth, we first calculate the global industry growth rate, $\dot{b}_i$, as the rate of growth for each industry's total (summed across all locations):

$$\dot{b}_i = \frac{1}{t_1 - t_0} \log \left(\frac{\sum_l y_{il,t_1}}{\sum_l y_{il,t_0}} \right) \tag{4.3}$$

Likewise, we calculate the average location growth rate, $\dot{b}_l$, by adding up all the industries in each location and calculating the location growth rate:

$$\dot{b}_l = \frac{1}{t_1 - t_0} \log \left(\frac{\sum_i y_{il,t_1}}{\sum_i y_{il,t_0}} \right) \tag{4.4}$$

Note that these variables would account for all the variance in growth rates of the industry-country cell if all industries within a country grew at the same rate or if all countries maintained their industry market share in the world. Deviations from balanced location growth mean that some industries are increasing or decreasing their share in the location's exports. Deviations from the radial industry growth mean that countries are changing their global market share in that industry. We use these radial growth variables for multiple purposes. First, they are an intuitive benchmark comparator for our density indices, as they represent an alternative theory of growth dynamics (balanced growth). Second, they are also useful to determine to what extent the density variables related to implied comparative advantage are capturing a dynamic that is orthogonal to balanced radial growth. Finally, we note that Equation 1.1, when using the RCA functional form, implies that

$$\dot{y}_{il} = \dot{r}_{il} - \dot{b}_i - \dot{b}_l + \dot{W}$$

where as before the dot operator denotes the changes in each logged variable between initial and final period. Specifically, $\dot{r}_{il}$ is the change in RpCA; this would mean we are then evaluating our residual densities' ability to predict the change in output intensity over time. Nevertheless, it is important to note that benchmarking the performance of the density residuals against radial growth is not a fair comparison, since the density variables are calculated with only base year data to explain growth, while the radial growth variables use information from the final period as well.

Column 6 shows the effect of radial growth and initial size variables on subsequent

growth. As expected, they are all statistically significant and economically meaningful. Column 7 includes these variables together with the density variables. The latter substantially maintain their economic and statistical significance while they increase the R^2 relative to column 6 by over nine percentage points.

In addition to the radial growth variables, we also test our model using industry and location fixed effects. These capture all industry and location effects, subsuming the size and radial growth control variables as well as any other source of variation at the purely location level or industry level. Thus, any additional explanatory power after controlling for the initial size of the industry-country cell and these fixed effects must come entirely from industry-location interactions.

Column 8 shows the baseline growth equation with both location and industry fixed effects as well as the initial location-industry size. Column 9 reintroduces the density variables and shows that their economic and statistical significance is undiminished. It is important to again point out that the product and country space residuals can be calculated using only base year data and thus contain no information regarding future growth, while the coefficients on the fixed effects can only be calculated ex post. This means that the residuals of the first-stage density regressions still carry new information related to industry-location growth in the subsequent 21 years, even after controlling for all possible industry and location effects.

Finally, it should also be pointed out that the robust and negative signs in the Columns 4-8 for initial industry-location exports confirm Rodrik (2013)'s observation of unconditional convergence at the industry level. But the significance of our density measures imply a richer structure in the convergence patterns of countries.

Next, we apply the same process to our US, Chile and India datasets, over the maximum period for which we have data (Table 7). We find that the product space and country space residuals are highly significant predictors of industry-location growth, both before and after controlling for initial output, industry and location size, and radial growth ($p < 0.01$ for all cases).

Table 7: OLS regression of employment, payroll and establishments growth by industry-location.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	USA, 2003-2011						Chile, 2005-2008		India, 1990-2005	
	Employment growth		Establishments growth		Payroll growth		Establishments growth		Employment growth	
Residual, Product Space density	-0.018*** (0.001)	-0.027*** (0.001)	-0.007*** (0.001)	-0.002*** (0.001)	-0.012*** (0.001)	-0.045*** (0.001)	-0.027*** (0.001)	-0.031*** (0.005)	-0.209*** (0.008)	-0.264*** (0.009)
Residual, Country Space density	-0.026*** (0.001)	-0.025*** (0.001)	-0.022*** (0.001)	-0.024*** (0.001)	-0.034*** (0.002)	-0.024*** (0.001)	-0.009*** (0.002)	-0.020*** (0.002)	-0.104*** (0.009)	-0.064*** (0.007)
Initial industry -location level (log)		0.008*** (0.001)		-0.003*** (0.000)		0.016*** (0.001)		0.007 (0.005)		-0.013 (0.008)
Initial location population (log)		-0.014*** (0.001)		0.003*** (0.000)		-0.026*** (0.002)		0.014*** (0.002)		-0.017 (0.012)
Initial global industry total (log)		-0.009*** (0.001)		-0.000 (0.000)		-0.023*** (0.002)		0.023*** (0.001)		0.039*** (0.009)
Radial industry growth (log)		0.383*** (0.004)		0.331*** (0.004)		0.379*** (0.004)		0.681*** (0.013)		1.047*** (0.010)
Radial location growth (log)		0.324*** (0.013)		0.283*** (0.009)		0.210*** (0.012)		0.423*** (0.042)		0.519*** (0.079)
Observations	279,439		278,946		89,378		50,373		49,651	
Adjusted R^2	0.150	0.213	0.092	0.226	0.162	0.340	0.059	0.310	0.199	0.427

Location-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

5 The extensive margin: Discrete industry appearances and disappearances

In previous sections we analyzed the rate of growth of exports, employment, payroll and number of establishments in industry-locations that already exist. In this section, we focus on the extensive margin, looking at the *appearance* and *disappearance* of industries in locations.

To do this, we first need to establish which industry-locations are present and which are absent. The case is simple when using the US and Chilean datasets because they report the number of establishments. In these cases, an industry is present in a location if at least one establishment is reported to exist there. Formally, we capture this signal with the binary presence variable M_{il}:

$$M_{il,t_0} = \begin{cases} 1 & y_{il,t_0} \geq 1 \\ 0 & y_{il,t_0} = 0 \end{cases} \tag{5.1}$$

where, as before, y_{il,t_0} is the number of establishments in industry i and location l in year t_0. In this notation, we refer to an industry location as *present* when $M_{il,t_0} = 1$ and *absent* when $M_{il,t_0} = 0$. Likewise, an appearance between years t_0 and t_1 is defined as $M_{il,t_0} = 0 \rightarrow M_{il,t_1} = 1$, while a disappearance is defined as $M_{il,t_0} = 1 \rightarrow M_{il,t_1} = 0$.

To study the extensive margin in the international trade dataset we need to decide on an equivalent definition of presence and absence. Here, the concern is that the data may include errors that imply the presence of an industry when it is simply a case of small re-exports or clerical error. We define an industry to be absent in a location if $R_{il,t_0} < 0.05$, meaning that exports are less than 1/20th of the average per capita exports for the world. We will consider an industry to be present if R_{il} is above 0.25. We will define an appearance as a move from $R_{il,t_0} < 0.05$ to $R_{il,t_1} > 0.25$ and a disappearance as a move from $R_{il,t_0} > 0.25$ to $R_{il,t_0} < 0.05$ as originally used by Bustos et al. (2012). Thus, our definition of extensive margin change represents a fivefold increase or decrease in output around very low levels. While these thresholds are somewhat arbitrary, we obtain similar results using different thresholds.

Table 8: Probit regression of industry-location extensive margin, US, Chile and International

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	USA (establishments) Industry presences in 2003			Chile (establishments) Industry presences in 2005			International (exports) Industry presences in 1995		
Product Space density, initial year	0.266*** (0.001)		0.022*** (0.002)	1.191*** (0.005)		1.165*** (0.006)	0.397*** (0.006)		0.306*** (0.007)
Country Space density, initial year		0.795*** (0.004)	0.772*** (0.005)		0.939*** (0.005)	0.822*** (0.006)		0.348*** (0.004)	0.160*** (0.004)
All industry-locations		768,888			227,454			159,960	
Present industries		324,622			55,347			47,337	
Presence rate		42.22%			24.33%			29.59%	
Area Under the Curve	0.924	0.940	0.940	0.815	0.900	0.911	0.933	0.859	0.914
Pseudo R^2	0.341	0.493	0.495	0.357	0.193	0.454	0.353	0.226	0.376

Location-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

We apply these definitions to the US and Chilean establishment data and to the international trade data. In the US, we classify 324,622 industry locations as present in 2003, or 42% of the total sample of industry locations. Of these present industries, 45,108 became absent by 2011, yielding a disappearance rate of 14%. Likewise, 37,681 industries that were absent in 2003 became present by 2011, resulting in an appearance rate of 8.5%. In Chile, 55,347 industries were present in 2005, or 24% of the sample. By 2008, 4,762 of these industries became absent (a disappearance rate of 8.6%) while 11,496 initially absent industries became present (an appearance rate of 6.7%). Internationally, 47,337 industries were present in our base year of 1995, or 29.6% of the sample. By 2010, 7,089 of these present industries became absent (a disappearance rate of 7.5%) while 3,648 initially absent industries became present (an appearance rate of 7.7%).

We can now use our density indices for the implied comparative advantage to explain the appearance and disappearance of industries by location. First, we use our density variables to generate an expected presence or absence estimation for each industry-location cell by using a probit model. In particular, we regress M_{il} on product space and country space density. Our probit model estimates the probability of industry presence in a location in the base year:

$$P\left(M_{il,t_0}=1\right)=\Phi\left(\alpha+\beta_{PS}w_{il}^{[PS]}+\beta_{CS}w_{il}^{[CS]}\right) \tag{5.2}$$

where Φ is a normal cumulative distribution function. Note that as for the intensive margin, the model in Equation 5.2 uses only information from the base year. Going forward, we denote the expected presence or absence of an industry in a location at time t_0 as M_{il,t_0}:

$$M_{il,t_0}=\widehat{M}_{il,t_0}+\varepsilon_{il,t_0} \tag{5.3}$$

where $\widehat{M}_{il,t_0}$ is the expected probability of industry presence and ε_{il,t_0} is the residual error term. We then use the residual to predict changes to M_{il,t_0}, i.e., industry appearances and disappearances. Our predictive criterion is that M_{il,t_0} will approach $\widehat{M}_{il,t_0}$ as time passes, that is, M_{il,t_0} approaches the values that are signaled by the country space and product space densities.

Table 9: Probit regression of changes in industry-location extensive margin, US, Chile and international

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	USA (establishments) Industry appearances, 2003-11			Chile (establishments) Industry appearances, 2005-08			International (exports) Industry appearances, 1995-2010		
Residual, Product Space density	-2.858*** (0.026)			-2.636*** (0.037)			-1.903*** (0.059)		
Residual, Country Space density		-3.004*** (0.017)			-1.757*** (0.038)			-1.327*** (0.032)	
Residual, hybrid density			-2.994*** (0.017)			-2.389*** (0.031)			-1.786*** (0.044)
Initially absent		444,266			172,107			94,547	
Industry appearances		37,681			11,496			7,089	
Appearance rate		8.48%			6.68%			7.50%	
Area under the curve	0.801	0.832	0.834	0.757	0.747	0.803	0.750	0.692	0.723
Pseudo R^2	0.059	0.145	0.144	0.064	0.021	0.073	0.019	0.027	0.028
	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
	USA (establishments) Industry disappearances, 03-11			Chile (establishments) Industry disappearances, 05-08			International (exports) Industry disappearances, 1995-2010		
Residual, Product Space density	2.953*** (0.018)			0.929*** (0.039)			1.213*** (0.032)		
Residual, Country Space density		2.265*** (0.009)			1.435*** (0.038)			1.630*** (0.050)	
Residual, hybrid density			2.272*** (0.009)			1.368*** (0.030)			1.265*** (0.031)
Initially present		324,622			55,347			47,337	
Industry disappearances		45,108			4,762			3,648	
Disappearance rate		13.90%			8.60%			7.71%	
Area under the curve	0.840	0.854	0.855	0.625	0.708	0.722	0.746	0.716	0.742
Pseudo R^2	0.231	0.249	0.250	0.022	0.066	0.081	0.080	0.068	0.087

Location-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

In addition to the pseudo-R^2 statistic, we evaluate the accuracy of these predictions using the *area under the receiver-operating characteristic (ROC) curve*. The ROC curve plots the rate of true positives of a continuous prediction criterion (the residual ε_{il,t_0} in our case) as a function of the rate of false positives. The area under the curve (AUC) statistic is equivalent to the Mann-Whitney statistic (the probability of ranking a true positive ahead of a false positive in a prediction criterion). By definition, a random prediction will find true positives and false positives at the same rate, and hence will result in an $AUC = 0.5$. A perfect prediction, on the other hand, will find all true positives before giving any false positive, resulting in an $AUC = 1$.

Table 8 applies our probit regression model to the US and Chilean establishment data and international export data to the first year for which we have information in the respective datasets. In the initial regression, we see that our product space and country space density terms explain between one third and one half of the variance in industry-location. Also, coefficients on all terms are positive and highly significant, meaning that a high value for density is strongly indicative of the presence of an industry in a location. The AUC are very high (AUC between 91% and 94% for hybrid models).

Next, we use the residual term from these regressions to predict industry appearances and disappearances over long-term horizons in each dataset (Table 9). For all cases, the coefficients are highly significant, and have the expected sign. In the US, over an 8-year period, the hybrid model predicts industry appearances with an AUC of 83% and disappearances with an AUC of 86%. For the Chilean data over a 3-year horizon, the hybrid model's AUC is 80% for appearances and 72% for disappearances. For the international trade data over a 15-year horizon, the AUC is 72.3% for appearances and 74.2% for disappearances. This suggests that the "unexpectedly absent" industries tend to preferentially appear over time while the "unexpectedly present" industries tend to disappear.

6 Robustness Checks

6.1 Choice of base year and end year

In our growth regression models, we chose the base year to be the initial year of the available data and the end year as the final year of the available data. We test the sensitivity of our results reported in Table 6 depending on the choice of different base years and end years. Figure 2 shows the adjusted R^2 values for our international trade regressions (including the base-year controls) over all possible year combinations. Each regression explains a sizable portion of the variation in export growth, with the lowest adjusted

R^2 exceeding 8.6%, and a mean R^2 of 14.8%. Interestingly, we find that predictive power appears to generally improve as the prediction interval increases (barring a possible continuity break between 1999 and 2000). This indicates that the density indices do not capture a short-term mean reversion effect, but a longer-term shift in economic structure.

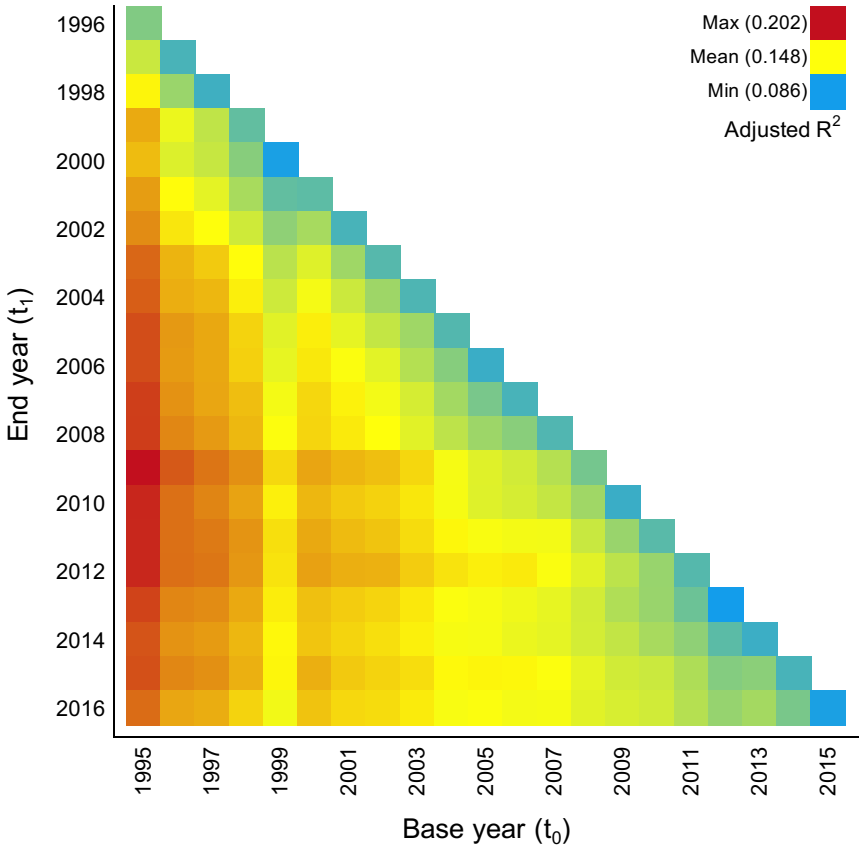


Figure 2: *Heat map of out of sample predictions of export growth, hybrid density model.*

6.2 Product and Country Groups

We can also ask if this theoretical framework is more powerful within certain subsets of industries or locations. We begin by calculating the similarities and densities as before, using the full set of the 1995 export data. Next, we calculate the stage one and two regressions as before (and with the base-year and radial growth controls), but restricted only to the subsamples. We then report the resulting adjusted R^2. That is, our results for the wood products subsample measures how well our standard density variables can predict export growth for goods in the wood products category alone.

Table 10 shows the results. For product subsamples (based on the HS chapters), it

Table 10: Cross-sectional Regressions for Product and Country Groups.

Products		Countries	
HS Chapter	Adjusted R^2	By income	Adjusted R^2
Electronics, Machinery & Equipment	0.449	Low income	0.379
		Upper middle income	0.355
Medical, Consumer & Other	0.435	High income	0.327
Plastics & Rubbers	0.393	Lower middle income	0.296
Processed Metals	0.326		
Processed Stone & Glass	0.323	By region	Adjusted R^2
Chemicals & Related	0.319	North America	0.427
Apparel & Textiles	0.302	East Asia & Pacific	0.414
Wood Products	0.299	Europe & Central Asia	0.398
Automotive, Planes, Ships & Related	0.292	South Asia	0.308
		Middle East & North Africa	0.266
Processed Foodstuffs	0.285	Latin America & Caribbean	0.259
Agricultural Products	0.251	Sub-Saharan Africa	0.255
Extractives	0.202		

appears that the most easily-explained categories are "high-tech" goods like electronics or medical devices. The worst predictions are in the extractives and agricultural categories; this makes sense, since shifts in these commodities may have more to do with geographic luck (e.g. oilfield discoveries) than shared technological requirements. Next, we can divide countries by income level (according to World Bank Group classifications): the groups are relatively close to each other, though low-income countries are the most predictable under our framework (possibly because they are less likely to shift their comparative advantage over the period). Finally, the results by region appear to fall roughly in order of (non-oil) income (unlike looking at income directly); this would make Latin America and the Caribbean somewhat less predictable than expected based on income alone.

6.3 Double-out-of sample robustness check

In order to calculate our density variables (measuring implied comparative advantage), exclude the location or industry being proxied from the weighted average. However, other information regarding that location or that industry is also used in the calculations of the similarity matrices. This may create some concerns regarding endogeneity. We can address this issue by splitting our data into a training set and a testing set, a process referred to as "cross validation" in the computer science literature. In this approach,

we build the density indices using only information found in the training set. For the product space, we estimate the similarity between industries using half of the locations. Likewise, for the country space, we estimate the similarity between locations using half of the industries. This approach leaves one quarter of the industry-location observations completely outside of the sets we used to build our similarity indices. Finally, we use these similarity indices to build density indices for the testing set. Having built our out-of-sample predictors, we can repeat the regressions using only the testing data.

Table 11: Out-of-sample OLS regression of international exports by industry-location, 1995.

	(1)	(2)	(3)
	Exports, 1995 (revealed comparative advantage, log)		
Product Space density (log) out-of-sample, 1995	0.916*** (0.025)		0.940*** (0.065)
Country Space density (log) out-of-sample, 1995	0.150*** (0.038)		0.063 (0.046)
Product Space density (log) in-sample, 1995		0.830*** (0.029)	-0.035 (0.066)
Country Space density (log) in-sample, 1995		0.357*** (0.049)	0.121** (0.053)
Adjusted R^2	0.622	0.557	0.622

$N = 23,794$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Tables 11 and 12 apply this process to our international trade dataset over the 1995-2010 period. We find that the explanatory power of our out-of-sample hybrid model is comparable to that of the in-sample model (R^2 values are 62.2% and 55.7% for regressions of current export levels, and 18.7% versus 18.5% for regressions of export growth). Furthermore, adding the in-sample density terms to the out-of-sample dataset yields a negligible marginal contribution to R^2. Finally, combining the in-sample and out-of-sample predictors shows a marginally higher R^2 but with drastically reduced significance, indicating a high degree of co-linearity between the two types of variables. This suggests that endogeneity is not driving our results.

Table 12: Out-of-sample OLS regression of growth in international exports by industry-location, 1995-2010

	(1)	(2)	(3)
	Growth in exports (log), 1995-2010		
Residual, Product Space density, out-of-sample, 1995	-0.012*** (0.002)		-0.006*** (0.002)
Residual, Country Space density, out-of-sample, 1995	-0.014*** (0.002)		-0.009** (0.004)
Residual, Product Space density, in-sample, 1995		-0.012*** (0.002)	-0.006*** (0.002)
Residual, Country Space density, in-sample, 1995		-0.014*** (0.002)	-0.006 (0.003)
Adjusted R^2	0.187	0.185	0.189

$N = 23,794$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

6.4 Excluding the same 2-digit industries in calculating densities.

We have thus far shown evidence that the intensity of an industry in a location can be proxied using the output of highly similar industries in that same location. However, our results could also be explained by how industries are defined: if a classification system arbitrarily splits a single economic activity into two industry categories, then we would expect to see them in similar intensities. In the classification of the international trade data, for example, we can see that HS6101 contains "Men's overcoats" whereas HS6102 contains "Women's overcoats." In fact, in Table 3, 7 out of 10 product pairs have the same 2-digit Harmonized System codes. To explore the possibility that our results are driven by such trivial cases, we calculate an adjusted product space density that excludes industries from same 2-digit Harmonized System categories; we then perform the same regressions as before.

Table 13 shows the result for the initial stage. As the table shows, the adjusted density is still highly significant, R^2 values decrease only slightly (from 62.7% to 60.7%), and with the coefficients of the standard and adjusted density terms are statistically indistinguishable (at the 5% level).

We also find little to no effect of this adjustment in the second stage: when we use the residuals from the first stage to predicting export growth between 1995 and 2016, excluding the same 2-digit industries from density does not hurt the predictive power.

Table 13: OLS regression of international exports by industry-location, 1995 excluding the same 2-digit industries

	(1)	(2)
	Exports, 1995 (Revealed Comparative Advantage, log)	
PS density (all products)	0.962***	
1995 (log)	(0.013)	
PS density (excluding same 2-digit category)		0.944***
1995 (log)		(0.014)
Adjusted R^2	0.627	0.607

$N = 94,046$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Comparing Columns 1 to 2, 3 to 4, 5 to 6 and 7 to 8 of Table 14, the coefficients of the residual terms are not statistically different from each other at 5% level. Moreover, the explanatory power, which is captured by the R^2 term, is not affected by excluding the same 2-digit products (decreases under 0.01). These results thus appear to reject the possibility that our findings are an artifact of the industry classification used.

7 Conclusions

In this paper we have shown that the intensity of an industry-location cell follows a pattern that can be discerned from the presence of related industries in that location (product-space density) or of that industry in related locations (country-space density). Moreover, the error term in the predicted pattern is not pure noise but instead carries information regarding the future level, and hence the growth rate, of that industry-location cell. These dynamics include components that are orthogonal to pure industry or location effects, but instead capture industry-location interactions. These results can be found using international trade data as well as sub-national data for the USA, India and Chile. We have shown evidence that they operate both at the intensive as well as the extensive margin, that they are not due to endogeneity in the information and that they operate most intensely at long horizons of over a decade.

Table 14: OLS regression of export growth of an industry in a country excluding the same 2-digit industries

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Growth in exports (log), 1995-2016							
Residual, PS density (all products)	-0.023***		-0.017***		-0.022***			
1995	(0.001)		(0.001)		(0.001)			
Residual, PS density (excluding same 2-digit)		-0.022***		-0.016***		-0.022***		
1995		(0.001)		(0.001)		(0.001)		
Industry-location exports			-0.006***	-0.006***	-0.001	-0.001	-0.004***	-0.005***
1995 (log)			(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Location population			0.007**	0.007**	0.004**	0.004*		
1995 (log)			(0.003)	(0.003)	(0.002)	(0.002)		
Global industry total, 1995 (log)			0.009***	0.009***	0.004***	0.003***		
1995 (log)			(0.001)	(0.001)	(0.001)	(0.001)		
Radial industry growth					0.981***	0.978***		
1995-2016 (log)					(0.018)	(0.018)		
Radial location growth					0.964***	0.968***		
1995-2016 (log)					(0.110)	(0.111)		
Industry FE							Yes	Yes
Location FE							Yes	Yes
Adjusted R^2	0.151	0.149	0.172	0.169	0.319	0.316	0.431	0.426

$N = 94,046$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Our methodology successfully combines the methods from international trade and relatedness-based measures often used in the regional development. We motivated our approach with a modified Ricardian model in which the industry productivity parameters of each location are correlated among technologically similar industries. This means that whatever determines the comparative advantage of a location in an industry also affects technologically related industries. In this case, product space density is informative of the advantage of a location in technologically related industries while country space density is informative of the presence of the industry in technologically similar locations. We can use these density variables to estimate an implied comparative advantage value. This information can be obtained even if the location does not currently host the industry.

An important question is why is it that our two density variables would carry information about the future level of the industry, even after controlling for location and industry effects and the overall growth rate of the location and the industry in question. One interpretation is that each industry-location cell is affected by a zero-mean independent and identically distributed (i.i.d.) shock that causes a deviation of its output from their equilibrium levels. In this interpretation, since over time the expected value of the i.i.d. shock is zero, then the underlying fundamentals become expressed and it is these that are captured by our approach. An alternative interpretation is that what we are observing is the consequence of inter-industry spillovers such as Marshallian and/or Jacobs externalities (Glaeser et al., 1992; Ellison et al., 2010; Beaudry and Schiffauerova, 2009). In this case, the productivity of an industry-location cell is affected by the presence of related industries through spillovers. The fact that these take time is what would explain why our predictive power peaks at time periods of a decade or more. Future research would need to test for these alternative hypotheses.

Our results are also informative for models of unbalanced growth. Much of growth theory has been based on the exploration of solutions around a balanced growth path, but there has been a growing literature that tries to cope with structural transformation, along the Kuznets facts, i.e. the secular decline of agriculture in employment and output, the rising share of services and the inverted U shaped path of manufacturing. To cope with these features, some models use non-homothetic demand, as in a minimum level of food consumption or a hierarchy of needs. Other models use differential capital intensities across industries that are then rebalanced as capital deepens (Baumol, 1967; Acemoglu and Guerrieri, 2008). However, the stylized facts uncovered in this paper show a more subtle and fine-grained structure of predictable transformations. First, the structure is observable in exports and not just in employment and output, meaning that what drives these regularities is changes in supply rather than changes in domestic demand.

Secondly, the patterns we observe are too intricate to be determined by differences in capital intensity. So this paper suggests that at least some drivers of differential growth lie elsewhere.

From a Ricardian viewpoint, the conjecture would be that mastery of specific technologies affects the productivity of related industries, a feature that is not incorporated into current Ricardian models that productivity draws are completely random (Eaton and Kortum, 2002), or sector specific (Costinot et al., 2012). Efforts to improve on one industry's productivity spillover into other related industries. The unexploited aspects of technological relatedness are reflected in the difference between a country's output structure and the international norm. These differences get diminished over time as firms exploit technological spillovers.

Ricardian models are reduced-form models, where other elements are subsumed in the labor productivity parameters. In the Appendix, we show that we can motivate our approach also with a model with an indeterminate number of factors of production. From a factor based model point of view, the intensity of output in an industry-location cell should be related to the adequacy of the match between the factor requirements of the industry and the factor endowments of the location. Industries with similar factor requirements should be similarly present across locations while similarly endowed locations should host a similar suite of industries. Hence, the correlation between the intensity of presence of pairs of industries across all locations is informative of the similarity of their factor requirements while the correlation between output intensity of pairs of locations across all industries is informative of the similarity in their factor endowments.

From the perspective of factor-based models like Hecksher-Ohlin-Vanek model, the explanation requires an understanding of forces that affect the differential accumulation of multiple factors and not just their reallocation, which should happen at shorter time horizons. One conjecture is that the world is characterized by many factors of production that enter differentially in different industries with a complex set of complementarities. At any point in time, the endowment of the many factors is not consistent with an equalization of their rates of return, causing differential factor accumulation. Furthermore, given complementarity, the accumulation of one factor, in response to an initial disequilibrium will affect the return to other factors triggering further factor accumulation. The pattern of factor proportions that equalize returns is better reflected in the international average than in the country's own history. As a consequence, the output composition derived from the experience of others can be informative of the long-term trends in a particular country. Future research should test whether any of these conjectures hold true.

8 Appendix

8.1 Using RCA instead of RpCA.

We chose to use RpCA rather than more traditional RCA measure because RCA could be affected by the price movements in other industries. To be explicit, suppose that a country is a successful exporter of a product in year t_0 with $RCA > 1$. And suppose it is share remains the constant in the world in a later year, but because of the movements in the commodity prices, the country's total exports increase as a share of the world exports. Although nothing fundamental changes in the country's ability to make this product because of the price movements in other products the country's RCA becomes smaller.

As a robustness check, we developed all our measures with Balassa's RCA as well. Table 15 shows that both Product Space and Country Space based on RCA are significant predictors, yet, the explanatory powers of these variables are lower than their counterparts using RpCA in Table 4.

Table 15: OLS regression of international exports by industry-location using RCA based density, 1995

	(1)	(2)	(3)
	Exports, 1995 (Revealed Comparative Advantage, log)		
Product Space Density (log), 1995	0.967***		0.811***
	(0.013)		(0.019)
Country Space Density (log), 1995		0.726***	0.232***
		(0.038)	(0.021)
Adjusted R^2	0.416	0.258	0.432

$N = 93,984$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

When we do the growth regressions as reported in Table 6, the residual in the first stage performs equivalently with its RcPA counterpart (Table 16).

Table 16: OLS regression of export growth of an industry in a country using RCA based density

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Growth in exports (log), 1995-2016								
Residual, Product	-0.023***		-0.015***		-0.015***		-0.016***		-0.016***
Space density, 1995	(0.001)		(0.002)		(0.002)		(0.001)		(0.001)
Residual, Country		-0.020***	-0.009***		-0.003*		-0.009***		-0.009***
Space density, 1995		(0.001)	(0.002)		(0.002)		(0.002)		(0.001)
Industry-location				-0.012***	-0.005***	-0.011***	0.000	-0.021***	-0.003***
exports, 1995 (log)				(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Location population				0.012***	0.007**	0.011***	0.004**		
1995 (log)				(0.003)	(0.003)	(0.002)	(0.002)		
Global industry total				0.014***	0.009***	0.011***	0.003**		
1995 (log)				(0.001)	(0.001)	(0.001)	(0.001)		
Radial industry						0.928***	0.976***		
growth, 1995-2016 (log)						(0.019)	(0.017)		
Radial location						0.748***	0.982***		
growth, 1995-2016 (log)						(0.112)	(0.105)		
Industry FE								Yes	Yes
Location FE								Yes	Yes
Adjusted R^2	0.146	0.135	0.156	0.121	0.174	0.234	0.320	0.411	0.438

$N = 93,984$. Country-clustered robust standard errors in parentheses.
Significance given as *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

8.2 Calculation of Expected Similarity Coefficient

In this Technical Appendix, we will derive the expected similarity coefficient between two locations (products) given that the revealed comparative advantage of industry i in location l is:

$$r_{il} = 1 - 4d^2(\psi_i, \lambda_l) \tag{8.1}$$

where d is the shortest distance between independent and uniformly distributed ψ_i and λ_l parameters on a circle of perimeter 1. We can define the similarity $\phi_{ii'}$ between two industries i and i' given by

$$\phi_{ii'} = (1 + \text{corr}\{r_i, r_{i'}\})/2 \tag{8.2}$$

where corr is defined as

$$\text{corr}\{r_i, r_{i'}\} = \frac{\sum_l (r_{il} - \bar{r}_i)(r_{i'l} - \bar{r}_{i'})}{\sqrt{\sum_l (r_{il} - \bar{r}_i)^2 \sum_l (r_{i'l} - \bar{r}_{i'})^2}} \tag{8.3}$$

Since each ψ_i and λ_l are independently distributed, using law of large numbers, the sums in the correlation expressions can be converted to expectation values, namely:

$$\text{corr}\{r_i, r_{i'}\} = \frac{E[(r_{il} - \bar{r}_i)(r_{i'l} - \bar{r}_{i'})|\psi_i, \psi_{i'}]}{\sqrt{E[(r_{il} - \bar{r}_i)^2|\psi_i] E[(r_{i'l} - \bar{r}_{i'})^2|\psi_{i'}]}} \tag{8.4}$$

Since $\psi_i$and $\psi_{i'}$ are identical independently variables, the correlation becomes:

$$\text{corr}\{r_i, r_{i'}\} = \frac{E[(r_{il} - \bar{r}_i)(r_{i'l} - \bar{r}_{i'})|\psi_i, \psi_{i'}]}{E[(r_{il} - \bar{r}_i)^2|\psi_i]} \tag{8.5}$$

To make the calculations more tractable, if we use $\tilde{r}_{il} = (1 - r_{il})/4 = d^2(\psi_i, \lambda_l)$ instead of r_{il}, the similarity measure will remain the same. Using the identity:

$$E[(\tilde{r}_{il} - \bar{\tilde{r}}_i)^2|\psi_i] = E[\tilde{r}_{il}^2|\psi_i] - E^2[\tilde{r}_{il}|\psi_i] \tag{8.6}$$

we can calculate the denominator in Equation 8.5 using these separate terms. First,

$$E[\tilde{r}_{il}|\psi_i] = \int_0^1 d^2(\psi_i, \lambda_l) d\lambda_l = 2 \int_0^{1/2} y^2 dy = 2[y^3/3]_0^{1/2} = 1/12 \tag{8.7}$$

and

$$E[\tilde{r}_{il}^2|\psi_i] = \int_0^1 d^4(\psi_i, \lambda_l)d\lambda_l = 2\int_0^{1/2} y^4 dy = 2[y^5/5]_0^{1/2} = 1/80 \tag{8.8}$$

hence, the denominator in Equation 8.5 becomes:

$$E[(\tilde{r}_{il} - \bar{\tilde{r}}_i)^2|\psi_i] = \frac{1}{80} - \left(\frac{1}{12}\right)^2 = \frac{1}{180} \tag{8.9}$$

We can write the numerator in Equation 8.5 as:

$$\begin{aligned} E[(r_{il} - \bar{r}_i)(r_{i'l} - \bar{r}_{i'})|\psi_i, \psi_{i'}] &= \int_0^1 \left(d^2(\psi_i, \lambda_l) - \frac{1}{12}\right)\left(d^2(\psi_{i'}, \lambda_l) - \frac{1}{12}\right) d\lambda_l \\ &= \int_0^1 [d(\psi_i, \lambda_l)d(\psi_{i'}, \lambda_l)]^2 d\lambda_l - \frac{1}{144} \end{aligned} \tag{8.10}$$

To calculate the integral, we will measure all the distances on the circle relative to ψ_i. Let's define $\Delta_{ii'} \equiv d(\psi_i, \psi_{i'})$. We can write the integral in Equation 8.10 as

$$\begin{aligned} \int_0^1 [d(\psi_i, \lambda_l)d(\psi_{i'}, \lambda_l)]^2 d\lambda_l = &\int_0^{1/2} [y(y - \Delta_{ii'})]^2 dy \\ &+ \int_{1/2}^{1/2+\Delta_{ii'}} [(1-y)(y - \Delta_{ii'})]^2 dy \\ &+ \int_{1/2+\Delta_{ii'}}^1 [(1-y)(1-y+\Delta_{ii'})]^2 dy \end{aligned} \tag{8.11}$$

The first integral in Equation 8.12 is:

$$\int_0^{1/2} [y(y - \Delta_{ii'})]^2 dy = \frac{20\Delta_{ii'}^2 - 15\Delta_{ii'} + 3}{480}$$

The second integral in Equation 8.12 is:

$$\int_{1/2}^{1/2+\Delta_{ii'}} [(1-y)(y - \Delta_{ii'})]^2 dy = \frac{16\Delta_{ii'}^5 - 80\Delta_{ii'}^4 + 160\Delta_{ii'}^3 - 120\Delta_{ii'}^2 + 30\Delta_{ii'}}{480}$$

Finally, the third integral in Equation 8.12 is:

$$\int_{1/2+\Delta_{ii'}}^{1} [(1-y)(1-y+\Delta_{ii'})]^2 dy = \frac{-16\Delta_{ii'}^5 + 20\Delta_{ii'}^2 - 15\Delta_{ii'} + 3}{480}$$

Hence

$$\int_0^1 [d(\psi_i, \lambda_l) d(\psi_{i'}, \lambda_l)]^2 d\lambda_l = \frac{-80\Delta_{ii'}^4 + 160\Delta_{ii'}^3 - 80\Delta_{ii'}^2 + 6}{480} = \frac{1}{180} - \frac{1}{6}\left(\Delta_{ii'} - \Delta_{ii'}^2\right)^2 \tag{8.12}$$

Plugging back calculated numerator and denominator into Equation 8.5, we obtain:

$$\begin{aligned} \text{corr}\{r_i, r_{i'}\} &= \frac{E[(r_{il} - \bar{r}_i)(r_{i'l} - \bar{r}_{i'})|\psi_i, \psi_{i'}]}{E[(r_{il} - \bar{r}_i)^2|\psi_i]} = \frac{1/180 - \left(\Delta_{ii'} - \Delta_{ii'}^2\right)^2/6}{1/180} \\ &= 1 - 30\left(\Delta_{ii'} - \Delta_{ii'}^2\right)^2 = 1 - 30\left(d(\psi_i, \psi_{i'}) - d^2(\psi_i, \psi_{i'})\right)^2 \end{aligned} \tag{8.13}$$

Then the similarity between industries i and i' is:

$$\phi_{ii'} = (1 + \text{corr}\{r_i, r_{i'}\})/2 = 1 - 15\left(d(\psi_i, \psi_{i'}) - d^2(\psi_i, \psi_{i'})\right)^2 \tag{8.14}$$

8.3 Motivation Based on Factor Content of Production

8.3.1 Relation to the Heckscher-Ohlin Model

This paper is related to the controversy surrounding the Leontief Paradox which has been as a major handicap of the Hecksher-Ohlin trade models. For analytical tractability, economic models are often written with few factors of production and are then extended to see if the theorems derived in the simpler setting hold for an arbitrary number of factors. But to test theories empirically, it has been necessary to take a stand on the relevant factors of production in the world. In his seminal papers, Leontief found evidence against the Heckscher-Ohlin prediction that the basket of exports of a country should be intensive in the relatively more abundant factors (Leontief, 1953, 1956). He did so by decomposing the factor content into two factors: capital and labor. Testing a multi-factor world required an extension of the Heckscher-Ohlin model, derived by Vanek (1968). The question then

moved onto which factors to take into account when testing the theory empirically. [10] In most cases, it was not possible to list all factors related to the production and the tests were limited to the factors that can be measured. But these models have implications about the world that need not take a stand on what are the relevant factors of the world but can eschew that issue. The thought experiment above illustrates this idea. Products that have similar production functions should tend to be co-exported by different countries with similar intensities. Countries with similar factor endowments should tend to have similar export baskets. We can use these implications of the HOV model to estimate the missing data in our thought experiment.

In the HOV tradition, the factor endowments of a location determine which industries will be present there. To set up this model, we will make following standard HOV assumptions:

1. There is full employment of all factors in each location.
2. Factor prices are equalized across all locations.
3. All locations have access to the same technologies for all industries.
4. Production technologies exhibit constant returns to scale. Note that requirements 2-4 imply that there would be a fixed optimal combination of factor inputs to produce each output.

With these assumptions, we can write the full employment condition for all factors in all locations as a linear function:

$$AY = F \tag{8.15}$$

where

[10] This opened up a long literature on the relative factor content of trade (Antweiler and Trefler, 2002; Bowen et al., 1987; Conway, 2002; Davis et al., 1997; Davis and Weinstein, 2001; Deardorff, 1982; Debaere, 2003; Hakura, 2001; Helpman and Krugman, 1985; Leamer, 1980; Maskus and Nishioka, 2009; Reimer, 2006; Trefler, 1993, 1995; Trefler and Zhu, 2000, 2010; Zhu and Trefler, 2005). For example, Bowen et al. (1987) test it with 12 factors. Davis and Weinstein (2001) argue that HOV, "when modified to permit technical differences, a breakdown in factor price equalization, the existence of nontraded goods, and costs of trade, is consistent with data from ten OECD countries and a rest-of-world aggregate (p.1423). Clearly, all of these modifications can be construed as involving other factors, such as technological factors causing measured productivity differences, factors associated with geographic location and distance that affect transport cost, or factors that go into making nontraded goods that are used in the production of traded goods. Trefler and Zhu (2010) argue that there is a large class of different models that have the Vanek factor content prediction meaning that a test of the factor content of trade is not a test of any particular model.

- $A = N_f \times N_i$ is a matrix of factor inputs required to produce one unit of output in each industry.
- $Y = N_i \times N_l$ is a matrix where $r_{i,l}$ represents location l's output in industry i.
- $F = N_f \times N_l$ is a matrix where $F_{f,l}$ represents location l's endowments of factor f.

From an empirical point of view, we can only observe Y is the matrix of industry-location outputs. Empirically, we do not observe either the factor requirements of each industry A or factor endowments of each location F. In fact, we do not even have an exhaustive list of all factors. Following Equation 2.4 of Feenstra (2003), it is convenient to put the observable Y matrix on the left and leave the unobservable matrices on the right. In order to achieve this, we assume that $N_i = N_f$ and the A matrix is invertible. We define $B = A^{-1}$ such that $B \times A = I_{N_f}$, where I_{N_f} is the $N_f \times N_f$ identity matrix. The B matrix indicates how much output is generated by the employment of each factor in an industry. If we multiply both sides of Equation 8.15 by the B matrix, we obtain:

$$Y = BF \tag{8.16}$$

What can be inferred about the B and F matrices given that we can only observe matrix Y? Obviously, we will not be able to get information about individual elements of these matrices. Yet, we will show that the similarities in the factor requirements of two industries or the similarity between the factor endowments of two locations can be obtained from the information in the Y matrix. In subsections below, we first develop similarity measures between the factor requirements of pairs of industries and between the factor endowments of pairs of locations. This will prove instrumental for our purposes.

8.3.2 Similarities between the factor requirements of two industries

We will now derive a measure of input similarity of two industries, using Equation 8.16. We will assume that two industries, i and i', are similar if their associated row vectors in the B matrix, namely B_i and $B_{i'}$, are similar. Each element of the Y matrix can be written as:

$$r_{il} = \sum_f B_{if} F_{fl} \tag{8.17}$$

If we denote r_i and B_i as the row vectors of Y and B matrices, this equation can be rewritten in vector notation for all locations as:

$$r_i = B_i F \tag{8.18}$$

We will now calculate the covariance across all locations of a given industry. For this we first need to calculate the average production of each industry. Given Equation 8.18, average production of industry i can be calculated as:

$$\overline{r}_i = \frac{\sum_l r_{il}}{N_l} = \sum_f B_{if} \frac{\sum_l F_{fl}}{N_l} = \sum_f B_{if} \overline{F}_f \tag{8.19}$$

where $\overline{F}_f$ is the average presence of factor f across all locations. Subtracting the last two expressions from one another, we arrive at:

$$r_i - \overline{r}_i = B_i (F - \overline{F}) \tag{8.20}$$

where $\overline{F}$ is a $N_f \times N_l$ matrix that repeats in each row f the average endowment of the world in that factor $\overline{F}_f$. Using Equation 8.20, we can relate the observed covariance of the rows of the Y matrix to those of the unobserved B matrix:

$$(r_i - \overline{r}_i)(r_{i'} - \overline{r}_{i'})^t = B_i (F - \overline{F})(F - \overline{F})^t B_{i'}^t \tag{8.21}$$

$C \equiv (F - \overline{F})(F - \overline{F})^t$ matrix is the covariance matrix of rows of F matrix and, by definition, it is a square and symmetric matrix. The C matrix can be written as:

$$C = U \Sigma U^t \tag{8.22}$$

where U is a unitary matrix formed by the eigenvectors of C and Σ is a diagonal matrix whose elements are eigenvalues of C. If we define $\tilde{B}_i = B_i U$, then we can write the right hand side of Equation 8.23 as

$$(r_i - \overline{r}_i)(r_{i'} - \overline{r}_{i'})^t = \sum_f \tilde{B}_{if} \tilde{B}_{i'f}^t \sigma_f \tag{8.23}$$

where σ_f is the f^{th} (largest) eigenvalue of the covariance matrix, C. In one extreme, we can assume $\sigma_f = \sigma$ for all f. This would happen, for instance, If all rows of the F matrix are independently and identically distributed (i.i.d.). An interpretation of this assumption is that locations accumulate factors separately and independently. This assumption is unlikely to be true about the world but it simplifies our proof considerably; we give evidence of the generality of this approach in our simulations. Using this assumption, the right hand side becomes

$$\sum_f \tilde{B}_{if}\tilde{B}^t_{i'f}\sigma_f = \sigma\tilde{B}_i\tilde{B}^t_{i'} = \sigma B_i U U^t B^t_{i'} = \sigma B_i B^t_{i'} \tag{8.24}$$

Dividing both sides of Equation 8.24 by the standard deviation of r_i and $r_{i'}$, we can relate the correlation of the rows of the Y matrix to elements of the B matrix:

$$\text{corr}\{r_i, r_{i'}\} = \frac{(r_i - \bar{r}_i)(r_{i'} - \bar{r}_{i'})^t}{\sigma_{r_i}\sigma_{r_{i'}}} \approx \frac{\sigma}{\sigma_{r_i}\sigma_{r_{i'}}} B_i B^t_{i'} \tag{8.25}$$

where corr represents the Pearson correlation between vectors. Since this is a variable with a range $(-1, 1)$ we renormalize it to build a similarity metric between 0 and 1. Hence, we can estimate a measure of the similarity between the factor requirements of two industries, i and i':

$$\phi_{ii'} = (1 + \text{corr}\{r_i, r_{i'}\})/2 \tag{8.26}$$

Following Hausmann and Klinger (2006) and Hidalgo et al. (2007), we refer to this industry-industry similarity matrix as the product space.

8.3.3 Similarities between factor endowments of two locations

To quantify the similarities between the factor endowments of two locations, we will use an analogous approach. For two locations l and l', we would like to measure the similarity between their factor endowment vectors, F_l and $F_{l'}$. If we denote r_l and F_l as the l^{th} column vectors of Y and F matrices respectively, the output of a location is related to its factor endowments by:

$$r_l = BF_l \tag{8.27}$$

Note that our calculations in Section 2.1.1 can be replicated here because if we take the transposes of both sides in Equation 8.27, we will arrive to an expression similar to Equation 8.18. Assuming that the columns of B matrix are independently and identically distributed, we can write (akin to Equation 8.25):

$$\text{corr}\{r_l, r_{l'}\} = \frac{(r_l - \bar{r}_l)^t(r_{l'} - \bar{r}_{l'})}{\sigma_{r_l}\sigma_{r_{l'}}} \approx \frac{\sigma'}{\sigma_{r_l}\sigma_{r_{l'}}} F^t_l F_{l'} \tag{8.28}$$

where $\bar{r}_l$ is the average production of location l, σ_{r_l} is the standard deviation of r_l, σ' is the diagonal of the covariance matrix ($(B - \overline{B})^t(B - \overline{B}) \approx \sigma' I_{N_f}$). We renormalize the correlation to build a similarity metric between 0 and 1 by adding 1 and dividing by 2.

Hence, we can estimate a measure of the similarity between the factor endowments of two locations, l and l' as:

$$\phi_{ll'} = (1 + \text{corr}\{r_l, r_{l'}\})/2 \tag{8.29}$$

where corr represents the Pearson correlation between vectors, r_l and $r_{l'}$. Following Bahar et al. (2014), we refer to this location-location similarity matrix as the country space.

8.3.4 Scaling the matrices

Locations and industries differ greatly in size. It is often useful to normalize each location and each industry using, for example, the revealed comparative advantage (Balassa, 1964) or location quotient or the relative per capita output of each industry in each location. We can show that the correlations calculated over the normalized data have the same information regarding the input similarity of industries or the endowment similarity of locations. To show this, let us assume that we divide each industry by its relative size, s_i, and each location by its corresponding size, s_l. We define the $\widehat{r}$, $\widehat{A}$ and $\widehat{F}$ matrices such that $\widehat{r}_{il} = r_{il}/(s_i s_l)$, $\widehat{A}_{fi} = s_i A_{fi}$ and $\widehat{F}_{fl} = F_{fl}/s_l$ then

$$\widehat{A}\widehat{r} = \widehat{F} \tag{8.30}$$

All the previous results will follow in this renormalized space.

Unfortunately, for the world as a whole we do not have the production data for each industry in each country. The closest data source that we can readily obtain is data on country exports. Here we will show how by using the normalized version of the export dataset we can obtain a very good approximation to their production correlation counterparts. Production is the sum of locally consumed and exported portions of outputs of industries in that location. Mathematically, we can write this as:

$$r_{il} = X_{il} + C_{il} \tag{8.31}$$

where X_{il} represents net exports and C_{il} represents local consumption. Subtracting the mean output of the industry i in all locations we obtain:

$$r_i - \overline{r}_i = (X_i - \overline{X}_i) + (C_i - \overline{C}_i) \tag{8.32}$$

Assuming homothetic preferences worldwide, and normalizing each industry element by its size, we can assume that $C_i = \overline{C}_i$. Therefore, correlations of columns of Y can be inferred from correlations of columns of X. Similarly, we can also look at the column

vectors of Y and X:

$$r_l - \bar{r}_l = (X_l - \overline{X}_l) + (C_l - \overline{C}_l) \tag{8.33}$$

Again, assuming homothetic preferences worldwide, and normalizing each location by its size then each country would consume the same share of products, implying that $C_l = \overline{C}_l$. Consequently, correlations between the columns of Y can be inferred from the correlations between the columns of X.

8.4 Extended simulations

We illustrate how well density variables for implied comparative advantage based on the presence of related industries in the same row or the value of the same industry in related columns predict the value of each entry in the r_{il} matrix by simulating a toy model with 100 countries and 100 products and assume a uniform distribution of the ψ_i and the λ_l on the unit circle $\mathbb{U}$. In the toy model, we exactly know the underlying parameters; hence, we can experiment with the model choice parameters. First, we verify that our industry similarity index captures the distance between the factor requirements of industries, and that our location similarity index captures the distance between the factor endowments of locations. Next, we estimate how well our density measures predict the output of each industry-location. We will then study the impact of different neighborhood filters at different levels of noise.

We first use our variables for implied comparative advantage to estimate the intensity of output of each industry-location cell. To do this, we estimate the product space density of industry i in location l by calculating the weighted average of the intensities of the k most similar industries in location l with the weights being the similarity coefficients of each industry i' to industry i. We also calculate the country space density of industry i in location l by estimating the weighted average of the intensity of industry i across the k most similar locations. Setting $k = 50$ and iterating the simulation through 5,000 trials, we find that our hybrid density model (i.e., a regression including both industry density and location density) is a powerful predictor of industry-location output (mean $R^2 = 0.784$, with 95% confidence interval of $[0.7150.853]$ across all simulations). However, we need not fix the neighborhood filter at $k = 50$. In Figure 1, the uppermost line shows the effect of neighborhood size on the R^2. We see that the highest R^2 value is found at $k = 4$.

This result implies that it is possible to predict the value of any entry in the r_{il} matrix looking at the presence of related industries in the same row or the value of the same industry in related columns. This in itself is an interesting implication of our approach. But,

as we will show in Section 4, not only do the product space and country space densities perform well at predicting the r_{il} matrix, but more surprisingly, the errors in the relationship between actual and fitted values of the r_{il} matrix are predictive of future growth, both when looking at the intensive margin as well as the extensive margin. It is as if the rest of the matrix has more information about what the value of a cell should be than the cell itself and deviations from this expectation are corrected through subsequent growth or decline.

Finally, we can extend our simulation to examine the effect of noise in the observed output. Until now, we have assumed that the output of an industry-location, $\tilde{r}_{il}$, is determined solely is determined solely by the distance between the technological requirement of the industry ψ_i and the technological ability of the location λ_l. We can call this the equilibrium output. Let us assume instead that the output of each industry-location can deviate from this equilibrium value because of a disturbance term ε_{il} that is normally distributed. We will explore the possibility that the disturbance term enters either linearly or exponentially. As a result of these assumptions, we no longer observe the equilibrium output r_{il}, but instead observe only the current output, $\tilde{r}_{il}$.

Because the error term is not correlated across location or industry, we can expect that averaging our density index over several neighbors will reduce the effect of noise on our results. That is, we can achieve a better estimate of the noise-free output r_{il} by averaging the observed, noisy output $\tilde{r}_{il}$ of the most similar industries and locations, since the error in their output levels might cancel out. Our simulations confirm this hypothesis. We test three levels of noise in the output. Given that the standard deviation of r_{il} in our surrogate data is 1.994 (median value from 5,000 trials) we set the standard deviation of the noise term to 1, 2 and 4 which are, respectively half, the same or twice the standard deviation of r_{il}.

Now that the observed output incorporates an error component over the equilibrium output, the density variables are better estimates of the underlying fundamental parameters ψ_i and the λ_l than the parameters that would be inferred using the actual production. We illustrate this using a simulation of our toy model with 100 locations and 100 industries, where we now vary the standard deviation of the error term. We can then use the above formulas to calculate simulated output, proximities, and densities, setting $u = v = 50$. Figure 3 illustrates the explanatory power of our three density variables both for the additive and the exponential error models. We graph the correlation between observed output and equilibrium output as a measure of how well the model is able to implicitly capture the values of the fundamental variables ψ_i and the λ_l. When the error term has a standard deviation near zero, observed output is almost perfectly related to the

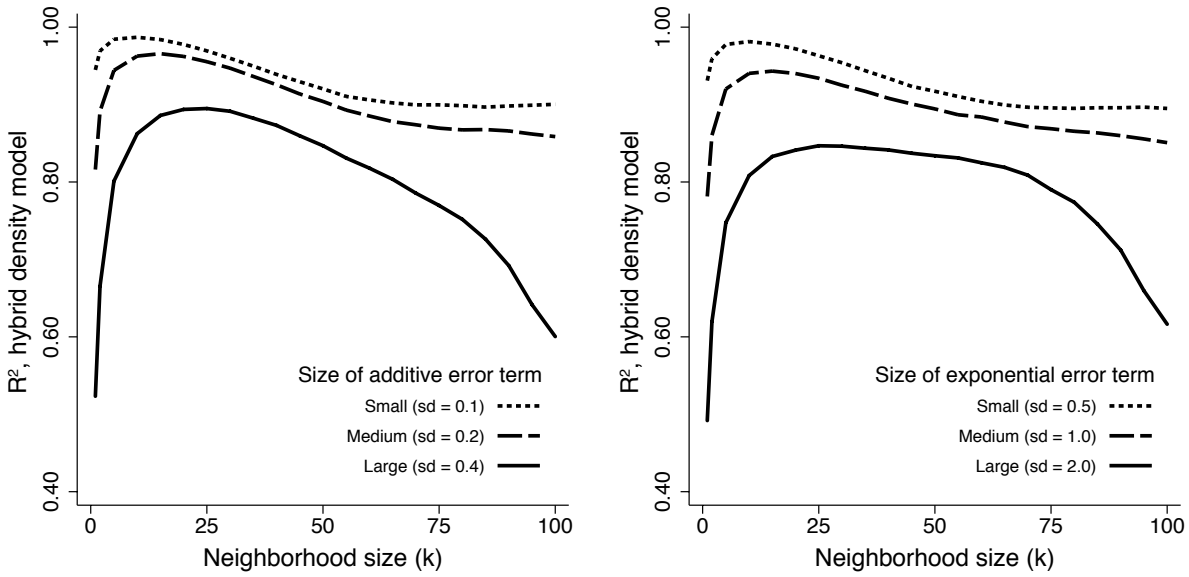


Figure 3: *Simulation of association between underlying output and hybrid density model, by size of neighborhood and noise level.*

underlying equilibrium output as estimated using the density variables. However, as the size of the error term increases, the observed output becomes increasingly less correlated with equilibrium output. The density variables are better able to capture the underlying structural variables and hence are better able to predict equilibrium output, with the Hybrid density outperforming either the product space or the country space densities because they average over a broader set of observations.

In Figure 1, we see the effect of increasing the size of the error term on the correlation between the density variables and the actual product intensity. First, we note that, as expected, a larger error term does reduce the R^2 of our estimates, though the decline is relatively small. Second, as noise increases, the R^2 peak tends to move toward mid-range k values, suggesting that the tradeoff between focusing on more related industries and averaging over a broader set of observations moves in favor of the latter. At the same time, the relationship between k and R^2 levels out as noise increases. For example, with a noise level of 2, the R^2 curve is fairly flat with predictive power roughly equal between k values of 4 and 150. When the neighborhood size gets larger, the predictive power decreases because the measure of density incorporates increasingly irrelevant information. This result suggests that finding the optimal neighborhood size may not be a first-order concern for our empirical tests.

8.4.1 Simulating the estimators on an HOV toy model

We test the effectiveness of our estimators of r_{il} by creating a surrogate dataset using a toy model based on our HOV model. First, we verify that our industry similarity index captures the distance between the factor requirements of industries, and that our location similarity index captures the distance between the factor endowments of locations. Next, we estimate how well our density measures predict the output of each industry-location. We will then study the impact of different neighborhood filters at different levels of noise.

To create our surrogate dataset, we set the number of industries N_i and the number of locations N_l both equal to 200. We also set the number of factors N_f equal to 200 to ensure that the A matrix is invertible. We then populate the A and F matrices using a uniform random distribution with values between zero and one. From these factor requirement and endowment matrices, we can produce a 200 by 200 matrix of output values r_{il} using the equation $Y = A^{-1}F$.

We can now explore whether the correlation between pairs of Y rows is related to the correlation between pairs of A^{-1} rows, meaning that the similarity of production or export intensity of products across all locations carries information about the similarity of their factor requirements, as indicated by Equation 8.25. We randomly select 5,000 A^{-1} and F matrices and test the validity of this equation. We note that the random selection of both matrices simultaneously puts no inherent structure into these matrices and in reality we expect to observe more structures matrices. Even in the random case, the correlation between the actual and estimated numbers exhibit is 0.532 ∓ 0.014. We also test whether the correlation between pairs of columns of Y is related to the correlation between the corresponding columns of factor endowments F as suggested by Equation 8.28 and obtained the same correlation coefficient. These results confirm that the correlations of rows (columns) in the Y matrix are informative about the correlation between rows in the A^{-1} matrix (columns in the F matrix). When we put more structure into the model by introducing higher order correlations in the A^{-1} matrix or the F matrix, our correlation coefficients increase significantly.

Next, we use our density index to estimate the intensity of output of each industry-location cell. To do this, we estimate the product space density of industry i in location l by calculating the weighted average of the intensities of the k most similar products in location l with the weights being the similarity coefficients of each industry to industry i. We also calculate the country space density of industry i in location l by estimating the weighted average of the intensity of industry i across the k most similar locations. Setting $k = 50$ and iterating the simulation through 5,000 trials, we find that our hybrid density model (i.e., a regression including both industry density and location density) is a power-

ful predictor of industry-location output (mean $R^2 = 0.784$, with 95% confidence interval of $[0.7150.853]$ across all simulations). However, we need not fix the neighborhood filter at $k = 50$. In Figure 4, the uppermost line shows the effect of neighborhood size on the R^2. We see that the highest R^2 value is found at $k = 4$.

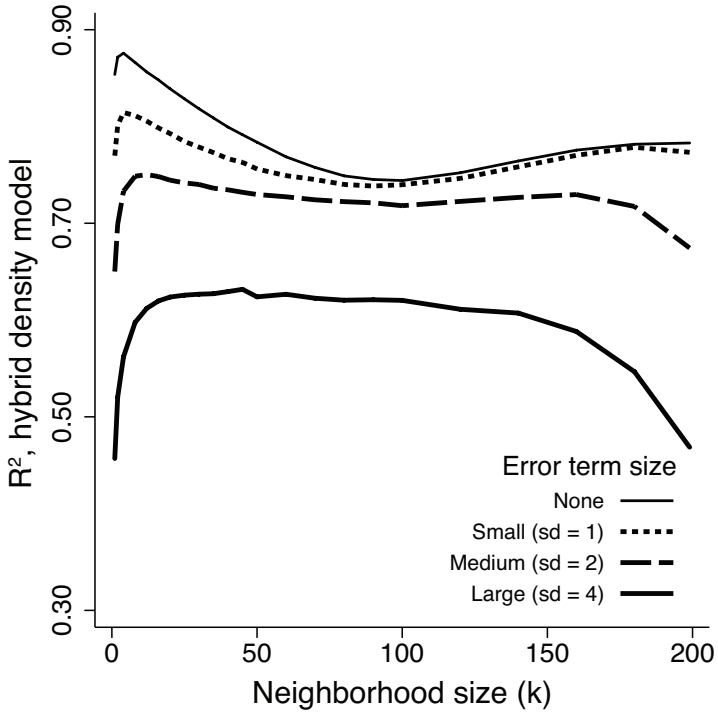


Figure 4: *Simulation of association between underlying output and hybrid density model, by size of neighborhood and noise level.*

Finally, we can extend our simulation to examine the effect of noise in the observed output. Beginning with the $Y = A^{-1}F$ used above, suppose that observed output, $\tilde{r}_{il}$, is affected by a random error term, ε_{il}, with a normal distribution around a mean of zero:

$$\tilde{r}_{il} = r_{il} + \varepsilon_{il} \tag{8.34}$$

Because the error term is not correlated across location or industry, we can expect that averaging our density index over several neighbors will reduce the effect of noise on our results. That is, we can achieve a better estimate of the noise-free output r_{il} by averaging the observed, noisy output $\tilde{r}_{il}$ of the most similar industries and locations, since the error in their output levels might cancel out. Our simulations confirm this hypothesis. We test three levels of noise in the output. Given that the standard deviation of r_{il} in our surrogate data is 1.994 (median value from 5,000 trials) we use assign the noise term standard deviations equal to 1, 2 and 4, which are approximately half, equal to and double

the standard deviation of r_{il}, respectively.

In Figure 4, we see the effect of increasing the size of the error term on the correlation between the density variables and the actual product intensity. First, we note that, as expected, a larger error term does reduce the R^2 of our estimates, though the decline is relatively small. Second, as noise increases, the R^2 peak tends to move toward mid-range k values, suggesting that the tradeoff between focusing on more related industries and averaging over a broader set of observations moves in favor of the latter. At the same time, the relationship between k and R^2 levels out as noise increases. For example, with a noise level of 2, the R^2 curve is fairly flat with predictive power roughly equal between k values of 4 and 150. This result suggests that finding the optimal neighborhood size may not be a first-order concern for our empirical tests.

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